James Ding 1/6/2023
Math EC

$$x^2 - x + a \;\Big|\; x^{13} + 0x^{12} + 0x^{11} + 0x^{10} + 0x^9 + 0x^8 + 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + x + 90$$

$$x^{11} + x^{10} + (1-a)x^9 + (1-2a)x^8 + (a^2-3a+1)x^7 + (3a^2-4a+1)x^6 + (-a^3+6a^2-5a+1)x^5 + (-4a^3+10a^2-6a+1)x^4 + (a^4-10a^3+$$

$$-(x^{13} - x^{12} + ax^{11})$$

$$x^{12} - ax^{11}$$

$$-(x^{12} - x^{11} + ax^{10})$$

$$(1-a)x^{11} - ax^{10}$$

$$-((1-a)x^{11} - (1-a)x^{10} + a(1-a)x^9)$$

$$(1-2a)x^{10} + a(1-a)x^9$$

$$(1-2a)x^{10} - (1-2a)x^9 + a(1-2a)x^8$$

$$(a^2-3a+1)x^9 + a(1-2a)x^8$$

$$(a^2-3a+1)x^9 - (a^2-3a+1)x^8 + a(a^2-3a+1)x^7$$

$$(3a^2-4a+1)x^8 + a(a^2-3a+1)x^7$$

$$(3a^2-4a+1)x^8 - (3a^2-4a+1)x^7 + a(3a^2-4a+1)x^6$$

$$(-a^3+6a^2-5a+1)x^7 + a(3a^2-4a+1)x^6$$

$$(-a^3+6a^2-5a+1)x^7 - (-a^3+6a^2-5a+1)x^6 + a(-a^3+6a^2-$$

$$(-4a^3+10a^2-6a+1)x^6 + a(-a^3+6a^2$$

$$(-4a^3+10a^2-6a+1)x^6 - (-4a^3+10a^2$$

$$(a^4-10a^3+$$

$$(a^4-10a^3+$$

$$(5a^4-$$

$$(5a^2-7a+1)x^3+(5a^4-20a^3+21a^2-8a+1)x^2+(-a^5+15a^4-35a^3+28a^2-9a+1)x-6a^5+35a^4-56a^3+36a^2-10a+1$$

$$(5a+1)x^5$$

$$-(5a+1)x^5$$

$$-6a+1)x^5+a(-4a^3+10a^2-6a+1)x^4$$

$$(5a^2-7a+1)x^5+a(-4a^3+10a^2-6a+1)x^4$$

$$(5a^2-7a+1)x^5-(a^4-10a^3+5a^2-7a+1)x^4+a(a^4-10a^3+5a^2-7a+1)x^3$$

$$10a^3+21a^2-8a+1)x^4+a(a^4-10a^3+5a^2-7a+1)x^3$$

$(5a^4 - 20a^3 + 21a^2 - 8a + 1)x^4 - (5a^4 - 20a^3 + 21a^2 - 8a + 1)x^3 + a(5a^4 - 20a^3 + 21a^2 - 8a + 1)x^2$

$(-a^5 + 15a^4 - 35a^3 + 28a^2 - 9a + 1)x^3 + a(5a^4 - 20a^3 + 21a^2 - 8a + 1)x^2$

$(-a^5 + 15a^4 - 35a^3 + 28a^2 - 9a + 1)x^3 + a(-a^5 + 15a^4 - 35a^3 + 28a^2 - 9a + 1)x^2 + a(-a^5 + 15a^4 - 35a^3 + 28a^2 - 9a + 1)x + 90$

$\therefore (-6a^5 + 35a^4 - 56a^3 + 36a^2 - 10a + 1) + a(-a^5 + 15a^4 - 35a^3 + 28a^2 - 9a + 1)x + 90$

$= x(a^6 - 21a^5 + 70a^4 - 84a^3 + 43a^2 - 11a + 2) + 6a^6 - 35a^5 + 56a^4 - 36a^3 + 10a^2 - a + 90$

Remainder $= (a-2)[a^5 - 19a^4 + 32a^3 - 20a^2 + 5a - 1]x + 6a^5 - 23a^4 + 10a^3 - 16a^2 - 22a - 45$

In order for $x^2 - x + a$ to divide evenly to $x^{13} + x + 90$, remainder $= 0$

$(a-2) = 0$

$\therefore \boxed{a = 2}$