

Differentiation

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:
 $f(0) = 2$, $f'(0) = -4$, and $f''(0) = 3$.

- (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show all work.

$$g(x) = e^{ax} + f(x)$$

$$g'(x) = e^{ax} \cdot \frac{d}{dx} ax + f'(x)$$

$$= ae^{ax} + f'(x)$$

$$g'(0) = ae^0 + f'(0)$$

$$\boxed{= a - 4}$$

$$g''(x) = ae^{ax} \cdot a + f''(x)$$

$$= a^2 e^{ax} + f''(x)$$

$$g''(0) = a^2 e^0 + f''(0)$$

$$\boxed{= a^2 + 3}$$

- (b) The function h is given by $h(x) = \cos(kx) \cdot f(x)$ for all real numbers and k constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x=0$.

$$h(x) = \cos(kx) \cdot f(x)$$

$$h'(x) = f(x) \cdot \frac{d}{dx} \cos(kx) + \cos(kx) \cdot f'(x)$$

$$= -\sin(kx) \cdot f(x) \cdot k + \cos(kx) \cdot f'(x)$$

$$h'(0) = -\sin(0) \cdot f(0) \cdot k + \cos(0) \cdot f'(0)$$

$$= 0 + 1 \cdot -4 = -4$$

$$h(0) = \cos(0) \cdot f(0) = 2$$

$$y - 2 = -4x$$