Area/Volume
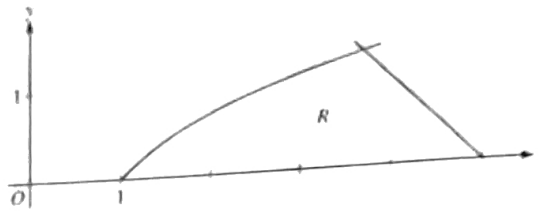

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

$y : 5 - \gamma$

a) Find the area of R.

Point of intersection: 3.693

$$\int_{1}^{3.693} \ln x \; dx \quad + \quad \int_{3.693}^{5} (5 - x) \; dx$$

$$= \quad 2.132 \quad + \quad 0.85 \quad \boxed{2.98}$$

b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

$$V = \int_{1}^{2.693} \left(\ln x \right)^2 \; dx \quad + \quad \int_{3.693}^{5} \left(5 - x \right)^2 \; dx$$

c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$\frac{2.98}{2} = \int_{0}^{k} \left((5-\gamma) \cdots e^{\gamma} \right) dy$$

$(\ln 5 - \gamma) - e^\gamma$

$y = k$

$$\boxed{\frac{2.98}{2} = \int_{0}^{k} \left((5 - y) - e^{y} \right) dy}$$