

Rates

The tide removes sand from Alameda Beach at a rate modeled by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$.

A pumping station adds sand to the beach at a rate modeled by $S(t) = \frac{15t}{1+3t}$.

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

- a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

$$\int_0^6 \left(2 + 5 \sin\left(\frac{4\pi t}{25}\right) \right) dt$$

= 31.8159 cubic yards

- b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t .

$$\int_0^t \left(2 + 5 \sin\left(\frac{4\pi x}{25}\right) + \frac{15x}{1+3x} \right) dx + 2500$$

- c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

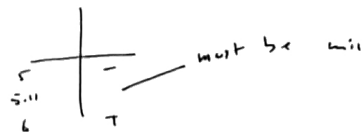
$$\left. \left(2 + 5 \sin\left(\frac{4\pi t}{25}\right) + \frac{15t}{1+3t} \right) \right|_{t=4} = 1.90$$

- d) For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify.

$$S(t) - R(t) = 0$$

$$s(t) = r(t)$$

$$\boxed{t = 5.11}$$



$$Y(t) = \int_0^t \left(\frac{15x}{1+3x} - \left(2 + 5 \sin\left(\frac{4\pi x}{25}\right) \right) \right) dx + 2500$$

= 2497.37