NAME James Ding
AP Calculus AB

DATE _____
FRQ #13

Rates

The tide removes sand from Alameda Beach at a rate modeled by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$.

A pumping station adds sand to the beach at a rate modeled by $S(t) = \frac{15t}{1+3t}$.

Both R(t) and S(t) have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

$$\int_0^6 \left(2 + 5\sin\left(\frac{4\pi t}{25}\right)\right) dt$$

$= 31.8159$ cubic yards

b) Write an expression for Y(t), the total number of cubic yards of sand on the beach at time t.

$$\int_0^t \left(\left(2 + 5\sin\left(\frac{4\pi x}{25}\right)\right) + \frac{15x}{1+3x}\right) dx + 2500$$

c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

$$-\left(2 + 5\sin\left(\frac{4\pi t}{25}\right)\right) + \frac{15t}{1+3t} \quad (t = 4) \approx -1.90$$

d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify.

$S(t) - R(t) = 0$

$S(t) = R(t)$

$t = 5.11$

must be min

$$Y(t) = \int_0^t \left(\frac{15x}{1+3x} + -\left(2 + 5\sin\left(\frac{4\pi x}{25}\right)\right)\right) dx + 2500$$

$= 2497.37$