NAME James Ding DATE ____________

AP Calculus AB FRQ #17

Rates

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

(a) What was the distance between the road and the edge of the water at the end of the storm?

$\int_0^5 (\sqrt{t} + \cos(t) - 3)\,dt + 35 = \boxed{26.49}$

(b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

After 4 hours, the rate ~~rate~~ at which the rate is changing at is ~~rate~~ 1.007 m/hr²

(c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

$0 \le t \le 5$

$\frac{1}{2\sqrt{x}} - \sin x = 0$ $x = 0.661,\ 2.84$

	f(t)
cp¹	−1.19
cp²	−2.26
ep 0	−2
5	−0.48

Decreasing most rapidly at $\boxed{t = 2.84}$

(d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

$-\int_0^5 f(t) = \int_0^x g(p)\,dp$

NAME James Ding DATE

AP Calculus AB FRQ #18

Rates

The rate at which people enter Disneyland on any given day is given by the function $E(t) = \frac{15600}{(t^2 - 24t + 160)}$.

The rate at which people leave Disneyland on the same day is given by the function $L(t) = \frac{9890}{(t^2 - 38t + 370)}$.

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which Disneyland is open. There are no people in the park right before it opens at $t = 9$.

(a) How many people have entered Disneyland by 5:00 pm? (Hint: what is t equal to at 5:00 pm?)

$$\int_{9}^{17} E(t)\,dt = \int_{9}^{17} \frac{15600}{t^2 - 24t + 160}\,dt \approx 6004.27$$

$$\boxed{\approx 6005}$$

(b) The price of admission is \$15 until 5:00 pm. After 5:00 pm, the price is \$11. How many dollars are collected from admission to the park?

$$15\int_{9}^{17} E(t) + 11\int_{17}^{23} E(t) = \boxed{104048}$$

(c) Let $H(t) = \int_9^t (E(x) - L(x))dx$ for $9 \le t \le 23$. $H(17) \approx 3725$. Find $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$.

H(17) describes the total # of people in the park at 5:00pm since 9

H'(17) describes the rate of ~~increase~~ change of people in the park at 5:00 since 9.

$$H'(t) = E(x) - L(x) = \frac{15600}{t^2 - 24t + 160} - \frac{9890}{t^2 - 38t + 370} = \boxed{-380.281}$$

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in Disneyland is a maximum?

$$0 = \frac{15600}{t^2 - 24t + 160} - \frac{9890}{t^2 - 38t + 370}$$

$$\boxed{t = 15.79}$$

NAME James Ding DATE

AP Calculus AB FRQ #19

In/Out Rates

On a typical day, the snow on a mountain melts at a rate modeled by the function

$$M(t) = \frac{\pi}{6}\sin\frac{\pi t}{12}.$$

A snow maker adds snow at a rate modeled by the function

$$S(t) = 0.006t^2 - 0.12t + 0.87.$$

Both M and S have units in inches per hour and t is measured in hours for $0 \le t \le 6$. At $t = 0$, the mountain has 40 inches of snow.

(a) How much snow will melt during the 6 hour period? Indicate units of measure.

$$\int_0^6 \frac{\pi}{6}\sin\left(\frac{\pi t}{12}\right)dt = 2 \text{ in}$$

(b) Write an expression for $I(t)$, the total number of inches of snow at any time t.

$$\cancel{I(t) = \int_0^t \left(\left[0.006a^2 - 0.12a + 0.87\right] - \left[\frac{\pi}{6}\sin\left(\frac{\pi a}{12}\right)\right]\right)da}$$

$$I(t) = \int_0^t \left(\left[0.006x^2 - 0.12x + 0.87\right] - \left[\frac{\pi}{6}\sin\left(\frac{\pi t}{12}\right)\right]\right)dx + 40$$

(c) Find the rate of change of the total amount of snow when $t = 3$.

$$s(3) - m(3) = 0.15336 \text{ in/hr}$$

(d) For $0 \le t \le 6$, at what time t is the amount of snow a maximum? What is the maximum value? Justify your answers.

$$s(x) - m(x) = 0 \;;\; 0 \le x \le 6$$

$x = 4.24$ ✓

4.23	+
4.24	0
4.25	−

Max

$$\boxed{\begin{array}{l} t = 4.24 \\ I(4.24) = 41.6517 \end{array}}$$