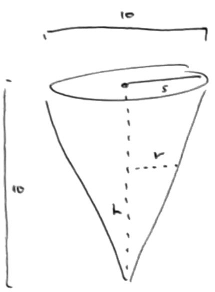
NAME ____________________ DATE ____________________

AP Calculus FRQ #2

Related Rates

A container has the shape of an open right circular cone. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr. Recall: The volume $V = \frac{1}{3}\pi r^2 h$, where h represents height and r represents radius.

(a) Find the volume of water in the container when h = 5 cm. Indicate units of measure.



$\frac{r}{h} = \frac{5}{10}$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 \cdot 5$

$\boxed{= \frac{125\pi}{12}\ \text{cm}^3}$

(b) Find the rate of change of the volume of water in the container, with respect to time, when h = 5 cm. Indicate units of measure.

Find $\frac{dV}{dt}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{\pi}{3}\left(\frac{h}{2}\right)^2 \cdot h$

$= \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$

$= \frac{\pi}{4}h^2 \cdot \frac{dh}{dt} = \frac{\pi}{4}(5)^2\left(-\frac{3}{10}\right) = -\frac{75\pi}{40} = -\frac{15\pi}{8}\ \frac{\text{cm}^3}{\text{hr}}$

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

$\frac{dV}{dt} = \frac{\pi}{4}h^2 \frac{dh}{dt}$

$= \frac{\pi}{4}h^2\left(-\frac{3}{10}\right)$

$= \frac{-3\pi}{40}h^2$

constant

$= -\frac{3\pi}{40}(2r)^2 = -\frac{3}{40}\pi(r^2)4 = -\frac{3}{10}\pi r^2$