

Tables

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

$$h(x) = f(g(x)) - 6$$

$$h(1) = 3$$

$$h(3) = -7$$

IUT. Because f and g are cont.

h must also be cont.

By IUT, there must

be a value r for $1 < r < 3$ s.t. $h(r) = -5$ because $-7 < -5 < 3$

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

h is cont. and differentiable. Conditions for MVT are met.

$$\frac{h(3) - h(1)}{3 - 1} = \frac{-7 - 3}{2} = \boxed{-5}$$

\therefore By MVT, there exists a value c for $1 < c < 3$ s.t. $h'(c) = -5$

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.

$$w'(x) = f(g(x)) \cdot g'(x)$$

at $x=3$

$$w'(3) = f(g(3)) \cdot g'(3) = -1 \cdot 2 = \boxed{-2}$$

(d) If g^{-1} is the inverse function of g , write an equation for the tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$$g^{-1}(x) = \frac{1}{g'(g^{-1}(x))}$$

$$g(1) = 2$$

$$g^{-1}(2) = 1$$

$$g^{-1}(g(1)) = g^{-1}(2)$$

$$= 1$$

$$\boxed{y^{-1} = \frac{1}{2}(x-2)}$$

Tables

1. The table below shows the behavior of a function f that is continuous for all real numbers. For the function, $f(2) = 4$, and $\lim_{x \rightarrow \infty} f(x) = 0$.

	$x < 4$	$x = 4$	$x > 4$
$f'(x)$	Positive	DNE	Negative
$f''(x)$	Negative	DNE	Positive

(a) For what values of x is f increasing? Explain.

$x \in (-\infty, 4)$. $f'(x)$ must be positive

(b) Does f have a relative maximum at $x = 4$? Justify your answer.

Yes. $f'(x) \rightarrow \text{DNE}$ at $x = 4$.
 Sign change from positive to neg.

(c) If possible, name the x -coordinate of the point of inflection on the graph of f . Justify your answer.

$x = 4$. f'' changes sign at $x = 4$

(d) Does MVT apply over the interval $[3, 5]$? Justify your answer.

No. Function must be differentiable. $f'(x)$ is DNE at $x = 4$.

(e) Sketch a possible graph of f .

