NAME James Ding   DATE ______________

AP Calculus   FRQ #27

Tables

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using units, interpret the meaning of your answer in the context of this problem.

$\frac{67.9 - 61.8}{15 - 9} = 1.017$

$W'(12)$ is equivalent to the average rate of change between $t \in (9,15)$, in °F/min

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$.

$\int_0^{20} W'(t)\,dt = W(20) - W(0) : 71 - 55 = 16$

Water temperature increases 16°F on $[0, 20]$

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$\frac{1}{20}\int_0^{20} W(t)\,dt = \frac{1}{20}\left[(4)(55) + (9-4)(57.1) + (15-9)(61.8) + (20-15)(67.9)\right]$

$= 60.79$

underestimate as it's an increasing function

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$W(25) = \int_{20}^{25} W'(t)\,dt + W(20)$

$= \int_{20}^{25} W'(t)\,dt + 71 = \boxed{73.043}$

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AP Calculus

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FRQ #28

Tables

Rocket A has positive velocity v(t) after being launched upward from an initial height of 0 feet at time t = 0 seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table.

t (sec)	0	10	20	30	40	50	60	70	80
$v(t)$ ft/sec	5	14	22	29	35	40	44	47	49

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

$$\frac{1}{b-a}\int_a^b A(t)\,dt$$

$$\frac{1}{80}\int_0^{80} A(t)\,dt = \frac{1}{80}\,v(t)\Big|_0^{80} = \frac{49-5}{80} = \frac{11}{20}\ \text{ft/sec}^2$$

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)dt$.

$\int_{10}^{70} v(t)dt$: Distance traveled by rocket on the interval $t \in (10,70]$

$$\int_{10}^{70} v(t)dt = 20\left[v(20) + v(40) + v(60)\right] = 20(22+35+44) = 2020\ \text{ft}$$

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time t = 0 seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet/second. Which of the two rockets is traveling faster at time t = 80 seconds? Explain.

$v_a(80) = 49$

$v_b(t) = \int \frac{3}{\sqrt{t+1}}\,dt = \int 3(t+1)^{-1/2}$

$= 6\sqrt{t+1} + C$; $v(0) = 2$

$C = -4$

$6\sqrt{t+1} - 4$

$v_b(80) \sim 50$

Rocket B is faster @ $t = 80$ seconds