NAME James Ding

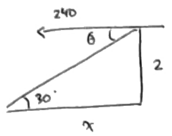
AP Calculus

DATE

FRQ #3

Differentiation Problems

1. A bombardier is sighting on a target on the ground directly ahead. If the bomber is flying 2 miles above the ground at 240 mph, how fast must the sighting instrument be turning when the angle between the path of the bomber and the line of sight is 30°?



$\tan\theta = \frac{2}{x}$

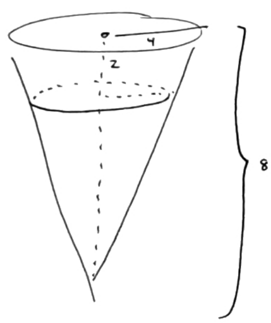
$\sec^2\theta \cdot \frac{d\theta}{dt} = -2x^{-2} \cdot \frac{dx}{dt}$

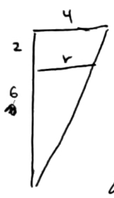
$\tan 30 = \frac{2}{x}$; $x = 2\sqrt{3}$

$\sec^2\frac{\pi}{6} \cdot \frac{d\theta}{dt} = -2(2\sqrt{3})^{-2} \cdot -240$

$\frac{d\theta}{dt} = \frac{-2 \cdot 240}{(2\sqrt{3})^2 \cdot \sec^2\frac{\pi}{6}} = \boxed{30 \text{ degrees/hour}}$

2. Water is running out of a conical funnel at the rate of 1 cubic inch per second. If the radius of the base of the funnel is 4 inches and the altitude is 8 inches, find the rate at which the water level is dropping when it is 2 inches from the top.





$\frac{4}{8} = \frac{r}{h}$; $\frac{h}{2} = r$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$

$1 = \frac{\pi}{4}(6)^2 \cdot \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{1}{9\pi} \text{ in/s}}$

3. Find the equations of the vertical lines which meet the curves $y = x^3 + 2x^2 - 4x + 5$ and $3y = 2x^3 + 9x^2 - 3x - 3$ in points at which the tangents to the respective curves are parallel.

$y = x^3 + 2x^2 - 4x + 5$

$y' = 3x^2 + 4x - 4$

$3y = 2x^3 + 9x^2 - 3x - 3$

$3\frac{dy}{dx} = 6x^2 + 18x - 3$

$\frac{dy}{dx} = 2x^2 + 6x - 1$

$3x^2 + 4x - 4 = 2x^2 + 6x - 1$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = -1, 3}$