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DATE

AP Calculus

FRQ #31

Differential Equation

Let f be a function with f(1) = 4 such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

a) Find the slope of the graph of f at the point where x = 1.

$\left.\frac{dy}{dx}\right]_{(1,4)}$ $\qquad \frac{3(1)+1}{2(4)} = \frac{1}{2}$

b) Write an equation for the line tangent to the graph of f at x = 1 and use it to approximate f (1.2).

$y-4=\frac{1}{2}(x-1)$ $\qquad y=\frac{1}{2}(x-1)+4$

$=\frac{1}{2}(1.2-1)+4$

$=4.1$

c) Find f(x) by solving the differential equation $\frac{dy}{dx}=\frac{3x^2+1}{2y}$ with the initial condition f(1) = 4.

$2y\,dy=(3x^2+1)\,dx$

$y^2+C=x^3+x+C$

$y^2=x^3+x+C$

$4^2=1^3+1+C$

$C=14$

$y=\pm\sqrt{x^3+x+14}$

$y=\sqrt{x^3+x+14}$

d) Use your solution from part (c) to find f(1.2).

$y=\sqrt{(1.2)^3+1.2+14}=4.114$