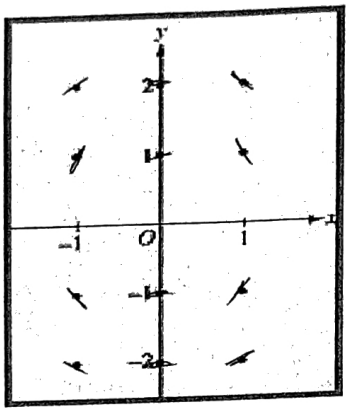
NAME _____ DATE _____

AP Calculus FRQ #33

Differential Equation

Given: $\frac{dy}{dx} = -\frac{2x}{y}$

a) On the graph provided, sketch a slope field for the given differential equation at the twelve points indicated.



b) Let y = f(x) be the particular solution to the differential equation with the initial equation f(1) = -1. Write an equation for the line tangent to graph of f at (1, −1) and use it to approximate $f(1.1)$.

$\frac{dy}{dx} = \frac{-2(1)}{-1} = 2$

$y+1 = 2(x-1)$

$y = -1 + 2(x-1)$

$y = -1 + (2)(0.1)$

$= -0.8$

c) Find the particular solution y = f(x) to the given differential equation with the initial condition f(1) = −1.

$\frac{dy}{dx} = \frac{-2x}{y}$

$\int y\,dy = \int -2x\,dx$

$\frac{1}{2}y^2 = -x^2 + C$

$\frac{1}{2} = -1 + C$

$C = \frac{3}{2}$

$\frac{1}{y}y^2 = -x^2 + \frac{3}{2}$

$y^2 = -2x^2 + 3$

$y = \pm\sqrt{-2x^2+3}$

$y = -\sqrt{2x^2+3}$