NAME James Ding DATE

AP Calculus FRQ #34

Differential Equation

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

$y - 1400 = mx$

$\frac{1}{25}(1400-300) = 44$

$W = 44t + 1400$

$W = 44(\frac{1}{4}) + 1400$

$\boxed{1411 \text{ tons}}$

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

$\frac{d^2W}{dt^2} = \frac{d}{dt}\left(\frac{1}{25}(W-300)\right)$

$= \frac{1}{25}\left(\frac{dW}{dt}\right)$

$= \frac{1}{25}\left(\frac{1}{25}(W-300)\right) = \frac{1}{625}(W-300)$

$> 0. \therefore$ CU

underestimate.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

$\frac{1}{W-300} dw = \frac{1}{5}$

$\frac{1}{W-300} dw = \frac{1}{25} dt$

$\int \frac{1}{W-300} du = \int \frac{1}{25} dt$

$\ln|W-300| = e^{\frac{1}{25}t + C} = e^{1/25} e^{C} = ke^{1/25}$

$1400 - 700 = ke^{0}$

$k = 1100$

$\boxed{W = 300 + 1100e^{\frac{1}{25}t}}$ $t \in (0,20)$

NAME [illegible] DATE

AP Calculus FRQ #35

Differential Equation

Given: $\frac{dy}{dx} = \frac{3-x}{y}$.

a) Let y = f(x) be the particular solution to the given differential equation for 1 < x < 5 such that the line y = −2 is tangent to the graph of *f*. Find the x-coordinate of the point of tangency, and determine whether *f* has a local max, local min, or neither at this point. Show justification of your work.

$0 = \frac{3-x}{y}$ $\boxed{x=3}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{3-x}{y}\right) = \frac{-y-(3-x)\frac{dy}{dx}}{y^2}$ $\boxed{\text{Min.}}$

$= \frac{-y}{y^2} = -\frac{1}{y} = \frac{1}{2}$ CU.

Let y = g(x) be the particular solution to the given differential equation for -2< x < 8, with the initial condition g(6) = -4. Find y = g(x).

$y\,dy = (3-x)\,dx$

$\int y\,dy = \int (3-x)\,dx$

$\frac{y^2}{2} = 3x - \frac{x^2}{2} + C$

$\frac{16}{2} = 18 - \frac{36}{2} + C$ $C = 8$

$\frac{y^2}{2} = 3x - \frac{x^2}{2} + 8$

$y = -\sqrt{6x - x^2 + 16}$