

The Particle Problem

For $0 \leq t \leq 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{\frac{t}{4}}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{\frac{t}{4}}\cos\left(e^{\frac{t}{4}}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

$$v(5.5) = 2\sin\left(e^{\frac{5.5}{4}}\right) + 1$$

$$= -0.453$$

$$a(5.5) = -1.2585$$

Speed is increasing, as it's increasing
 speeding up in the negative direction.

(b) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.

$$\frac{1}{6} \int_0^6 2\sin\left(e^{\frac{t}{4}}\right) + 1 dt = \boxed{1.949}$$

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

$$\int_0^6 |v(t)| dt$$

$$= \boxed{12.573}$$

(d) For $0 \leq t \leq 6$, the particle changes direction exactly once. Find the position of the particle at that time.

$$v(t) = 0$$

$$2\sin\left(e^{\frac{t}{4}}\right) + 1 = 0$$

$$t = 5.188$$

$$\int_0^{5.188} v(t) dt = \boxed{14.1547}$$