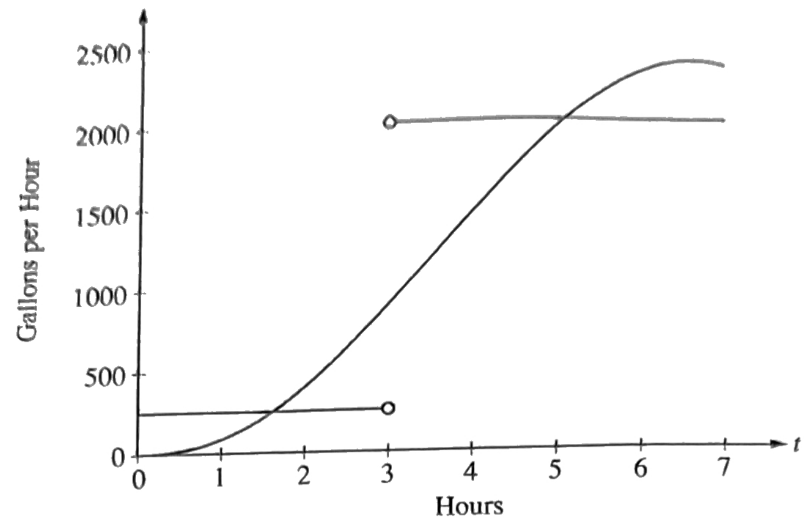
FRQ # 33



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

a. $\int_0^7 100t^2 \left(\sin\sqrt{t}\right) dt = \boxed{8264 \text{ gallons.}}$

b. $f(t) - g(t) < 0 \quad g(t) > f(t) \quad \therefore \boxed{(0, 1.617) \cup (3, 5.078)}$

$w(t) = 5000 + \int_0^t (f(x) - g(x))\,dx$

$w'(t) = f(t) - g(t) = 0$ / DNE.

$t = 1.617$ & 5.076, 3

min

max.

When $t = 3$, a max occurs. water ~~level~~ is at 5126.07 gallons.