NAME _____ DATE _____

AP Calculus AB FRQ #40

The Tangent Line

Consider the curve defined by $x^2 + 4xy + y^2 = -12$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

$2x + 4\left[y + x\frac{dy}{dx}\right] + 2y\frac{dy}{dx} = 0$

$2x + 4y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{(2x+4y)}{4x+2y} = -\frac{(x+2y)}{2x+y}$

(b) Find the equations of all horizontal tangent lines.

$\frac{dy}{dx} = \frac{-(x+2y)}{2x+y} = 0$

$x+2y=0$

$x=-2y$

$(-2y)^2 + 4y(-2y) + y^2 = -12$

$4y^2 - 8y^2 + y^2 = -12$

$y^2 = 4$

$y = \pm 2$

(c) Find the equation of the tangent line at the point (-4, 14).

$y - 14 = -4(x+4)$

$\frac{-(-4+2\cdot 14)}{2\cdot -4 + 14} = -4$

(d) If $\frac{dy}{dt} = \frac{-1}{2}$ at the point (-4, 14), find $\frac{dx}{dt}$.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{dx}{dt} = \frac{-1/2}{-4} = \boxed{\frac{1}{8}}$

(e) Use the tangent line in part c to estimate the value of k for the point (-4.01, k) on the curve.

$y - 14 = -4(x+4)$

$y = 14 - 4(x+4)$

$= 14 - 4(-4.01 + 4)$

$\boxed{\approx 14.04}$