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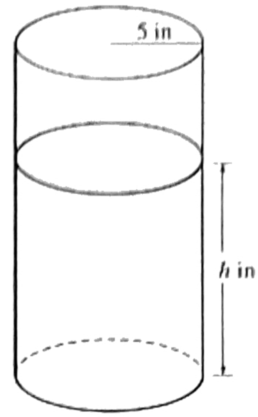
AP Calculus AB   FRQ #42

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

a. $V = \pi r^2 h$ $\qquad \frac{dV}{dt} = -5\pi\sqrt{h}$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$-5\pi\sqrt{h} = \pi r^2 \frac{dh}{dt}$

$-5\sqrt{h} = r^2 \frac{dh}{dt}$

$-5\sqrt{h} = 25 \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = -\frac{\sqrt{h}}{5}}$ ✓

b. $\frac{1}{\sqrt{h}}\,dh = -\frac{1}{5}\,dt$

$\int \frac{1}{\sqrt{h}}\,dh = -\int \frac{1}{5}\,dt$

$2\sqrt{h} = -\frac{1}{5}t + C$

$2\sqrt{17} = C$

$2\sqrt{h} = -\frac{1}{5}t + 2\sqrt{17}$

$\sqrt{h} = -\frac{1}{10}t + \sqrt{17}$

$h = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$

c. $0 = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$

$t = \boxed{10\sqrt{17}}$