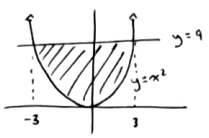
NAME James Ding
AP Calculus

DATE
FRQ #7

Area/Volume

A region is bounded by the graph of $y = x^2$ and the line $y = 9$.

(a) Find the area of this region.



$$\int_{-3}^{3} \left(9 - x^2\right) dx$$

$$= 2\int_{0}^{3} \left(9 - x^2\right) dx$$

$$= 2\left[9x - \frac{1}{3}x^3\right]_0^3 = 2\left[27 - \frac{1}{3}(27)\right] = 2\left[18\right] = \boxed{36}$$

(b) Find the volume of the solid generated by revolving this region about the x-axis.

2
27
3
81

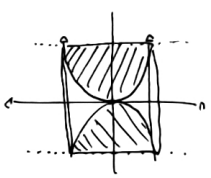
81
3
243

$$\pi\int_{-3}^{3} \left(9^2 - x^4\right) dx$$

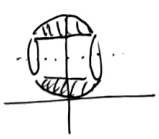
$$= 2\pi\int_{0}^{3} \left(81 - x^4\right) dx$$

$$= 2\pi\left[81x - \frac{1}{5}x^5\right]_0^3 dx$$

$$= 2\pi\left[243 - \frac{1}{5}(243)\right] = \boxed{\frac{1944\pi}{5}}$$



(c) There exists a number w, $w > 9$, such that when the region is revolved about the line $y = w$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of w.



$$\pi\int_{-3}^{3} \left((w - x^2) - (w - 9)^2\right) dx = \frac{1944\pi}{5}$$

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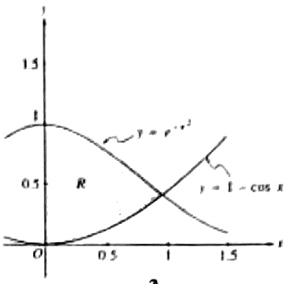
FRQ #8

Area/Volume

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis.



(a) Find the area of the region R.

Find intersection between $y = e^{-x^2}$ and $1 - \cos x$

$(0.942, 0.412)$

$$\int_0^{0.942} \left(e^{-x^2} - (1-\cos x) \right) dx = 0.590962$$

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

$$\int_0^{0.942} \left(\left(e^{-x^2}\right)^2 - (1-\cos x)^2 \right) dx = 1.74661$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$$\int_0^{0.942} \left(e^{-x^2} - (1-\cos x) \right)^2 dx$$

$$= 0.461064$$