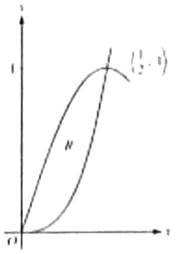
NAME James Ding

AP Calculus

DATE

FRQ #9

Area/Volume



Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

$f'(x) = 24x^2$

$f'\left(\frac{1}{2}\right) = 24\left(\frac{1}{2}\right)^2 = 6$

$f(x) = 8x^3$

$f\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^3$

$\boxed{y - 1 = 6\left(x - \frac{1}{2}\right)}$

(b) Find the area of R.

$\left(\frac{1}{2}, 1\right)$

$\int_0^{1/2} \left(\sin \pi x - 8x^3\right) dx = \left[-\frac{\cos \pi x}{\pi} - 2x^4\right]_0^{1/2}$

$\boxed{= \frac{1}{\pi} - \frac{1}{8} = 0.19331}$

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

$\boxed{V = \pi \int_0^{1/2} \left((1 - 8x^3)^2 - (1 - \sin \pi x)^2\right) dx}$

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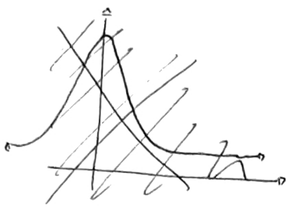
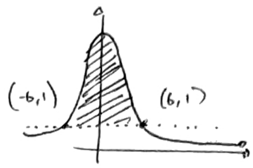
FRQ #10

Area/Volume

A region in the first and second quadrants is bounded above by the graph of $y = \frac{37}{1+x^2}$ and below by $y = 1$.

(a) Graph this region and find its area.





$$2\int_0^6 \left(\frac{37}{1+x^2} - 1\right) dx = 2\left[\tan^{-1}x \cdot 37 - x\right]_0^6$$

$$\boxed{= 92.0179}$$

(b) Find the volume of the solid generated by revolving this region about the x-axis.

$$\pi\int_{-6}^{6}\left(\left(\frac{37}{1+x^2}\right)^2 - 1^2\right) dx$$

$$= 2\pi\int_0^6\left(\left(\frac{37}{1+x^2}\right)^2 - 1\right) dx$$

$$\boxed{= 6705.2}$$

(c) The region is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$$A = \frac{1}{2}\pi r^2$$

$$= \frac{\pi}{2}\left(\frac{1}{2}\left(\frac{37}{1+x^2} - 1\right)\right)^2$$

$$V = \int_{-6}^{6}\left(\frac{\pi}{2}\left(\frac{1}{2}\left(\frac{37}{1+x^2} - 1\right)\right)^2\right) dx$$

$$\boxed{= 765.879}$$

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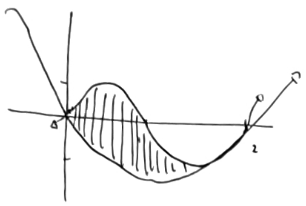
FRQ #11

Area/Volume

Let R be a region bounded by the graphs of $y = \sin(\pi x)$, $y = x^3 - 4x$, and $x = 2$. Consider only $x \geq 0$.

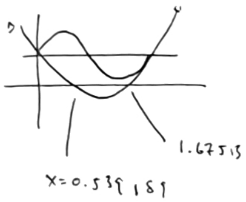
(a) Graph and find the area of R.



$$\int_0^2 \left(\sin \pi x - (x^3 - 4x)\right) dx$$

$$= \left[-\frac{\cos \pi x}{\pi} - \frac{1}{4}x^4 + 2x^2 \right]_0^2 = \left(-\frac{1}{\pi} - 4 + 8\right) - \left(-\frac{1}{\pi}\right) = \boxed{4}$$

(b) The horizontal line $y = -2$ splits the region R into parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.



$$\boxed{\int_{0.539181}^{1.67513} \left(-2 - (x^3 - 4x)\right) dx}$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$$V = \int_0^2 \left(\sin \pi x - (x^3 - 4x)\right)^2 dx = \boxed{9.97834}$$

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of the water in the pond.

$$\int_0^2 \left(\sin \pi x - (x^3 - 4x)\right)(3 - x) = \boxed{836495}$$