NAME ______________________ DATE ______________

AP Calculus AB

Derivative Exam Practice

1. Find $\frac{dy}{dx}$ for $x^3 + x^2y + 4y^2 = 6$

$\frac{d}{dx}\left[x^3 + x^2y + 4y^2\right] = 0$

$3x^2 + \left(x^2 \cdot \frac{dy}{dx} + y \cdot 2x\right) + 8y\frac{dy}{dx} = 0$

$x^2 \cdot \frac{dy}{dx} + 8y\frac{dy}{dx} = -3x^2 - 2xy$

$\boxed{\frac{dy}{dx} = -\frac{3x^2 + y^2x}{x^2 + 8y}}$

2. Differentiate implicitly $\sqrt{xy} = 1 + x^2y$

$\frac{d}{dx}\sqrt{xy} = \frac{d}{dx}\left(1 + x^2y\right)$

$\frac{1}{2}(xy)^{-1/2} \cdot \left(x\frac{dy}{dx} + y\right) = x^2 \cdot \frac{dy}{dx} + y \cdot 2x$

$\frac{x}{2\sqrt{xy}} \cdot \frac{dy}{dx} + \frac{y}{2\sqrt{xy}} = x^2\frac{dy}{dx} + 2xy$

$\frac{x}{2\sqrt{xy}} \cdot \frac{dy}{dx} - x^2 \cdot \frac{dy}{dx} = 2xy - \frac{y}{2\sqrt{xy}}$

$\frac{dy}{dx} = \frac{2xy - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - x^2}$

$\boxed{= \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}}$

3. Find the tangent to $x^3 + y^3 = 6xy$ at the point (3,3).

$\frac{d}{dx}\left[x^3 + y^3\right] = \frac{d}{dx}6xy$

$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 6\left(x \cdot \frac{dy}{dx} + y\right)$

$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 6x \cdot \frac{dy}{dx} + 6y$

$3y^2\frac{dy}{dx} - 6x \cdot \frac{dy}{dx} = 6y - 3x^2$

$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$

$\frac{6(3) - 3(3)^2}{3(3)^2 - 6(3)} = \frac{18 - 27}{27 - 18} = -1$

$\boxed{y - 3 = -(x - 3)}$

At what points on the curve is the tangent horizontal?

$\begin{cases} 6y - 3x^2 = 0 \\ x^3 + y^3 = 6xy \end{cases}$

$6y = 3x^2$

$y = \frac{1}{2}x^2$

$x^3 + \left(\frac{1}{2}x^2\right)^3 = 6x\left(\frac{1}{2}x^2\right)$

$x^3 + \frac{1}{8}x^6 = 3x^3$

$\frac{1}{8}x^6 - 2x^3 = 0$

$x^3\left(\frac{1}{8}x^3 - 2\right) = 0 \quad ; x = 0, y = 0$

$\frac{1}{8}x^3 - 2 = 0$

$\frac{1}{8}x^3 = 2$

$x^3 = 16$

$x = \sqrt[3]{16}$

$6y - 3x^2 = 0$

$6y - 3(\sqrt[3]{16})^2 = 0$

$y = 2^{5/3}$

$\boxed{\begin{matrix}(0,0) \\ (\sqrt[3]{16}, 2^{5/3})\end{matrix}}$

4. What is the derivative of $y = \sin^{-1}(x^2)$?

$\frac{d}{dx} \sin^{-1}(x^2)$

$\frac{1}{\sqrt{1-x^4}} \cdot 2x$

$\boxed{= \frac{2x}{\sqrt{1-x^4}}}$

5. Find the jerk of $y = 5x^3 - 6x^2 - 5x + 34$

$f'(x) = 15x^2 - 12x - 5$

$f''(x) = 30x - 12$

$f'''(x) = \boxed{30}$

6. Find y′′ if $x^4 + y^4 = 16$

$\frac{d}{dx}\left[x^4 + y^4\right] = 0$

$4x^3 + 4y^3 \cdot \frac{dy}{dx} = 0$

$4y^3 \cdot \frac{dy}{dx} = -4x^3$

$\frac{dy}{dx} = \frac{-4x^3}{4y^3}$

$= -\frac{x^3}{y^3}$

$\frac{d}{dx}\left(-\frac{x^3}{y^3}\right)$

$= -\frac{y^3 \cdot 3x^2 - x^3 \cdot 3y^2 \cdot \frac{dy}{dx}}{y^6}$

$= -\frac{3y^2x^2 - 3x^3y^2\left(-\frac{x^3}{y^3}\right)}{y^6}$

$= -\frac{3y^4x^2 + 3x^6}{y^7}$

$= -\frac{3x^2(y^4 + x^4)}{y^7}$

$\boxed{= -\frac{48x^2}{y^7}}$

7. Find $\frac{dy}{dx}$ for $y = x^x$

$\ln y = x \ln x$

$\frac{d}{dx} \ln y = \frac{d}{dx}\left[x \ln x\right)$

$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x$

$\frac{dy}{dx} = y\,(1 + \ln x)$

$\boxed{= x^x\,(1 + \ln x)}$

8. Find y' for $y = \log_a a^{\cos x}$

$y = \cos x \cdot \log_a a$

$= \cos x$

$\frac{d}{dx} \cos x = \boxed{-\sin x}$

9. Find an equation of the tangent line to the curve y = **ln ln x** at (e , 0).

$y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \cdot \frac{d}{dx} \ln x$

$= \frac{1}{x \ln x}$

$= \frac{1}{e}$

$\boxed{y = \frac{1}{e}(x - e)}$

10. Differentiate $f(x) = \ln|\sqrt{6x-1}\,(4x+5)^3|$

$\ln|\sqrt{6x-1}| + \ln|(4x+5)^3|$

$\frac{1}{2}\ln|6x-1| + 3\ln|4x+5|$

$\frac{d}{dx}\quad \frac{1}{2}\cdot\frac{1}{6x-1}\,6 + 3\cdot\frac{1}{4x+5}\cdot 4$

$$\boxed{\frac{3}{6x-1} + \frac{12}{4x+5}}$$

11. Differentiate $y = \dfrac{(\sin 2x)\,(\tan x)^3}{(x+2)^3}$

$\ln y = \ln\left(\sin 2x \cdot (\tan x)^3\right) - 3\ln(x+2)$

$= \ln(\sin 2x) + 3\ln(\tan x) - 3\ln(x+2)$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{\sin 2x}\cdot\cos 2x\cdot 2 + \frac{3}{\tan x}\cdot\sec^2 x$

$\frac{1}{y}\cdot\frac{dy}{dx} = 2\cot 2x + \frac{3}{\sin x\cos x}$

$$\boxed{\frac{dy}{dx} = \frac{\sin 2x\cdot\tan^3 x}{(x+2)^3}\left[2\cot 2x + \frac{3}{\sin x\cos x}\right]}$$

12. $x = 2\sqrt{4\sin y - 6\cos y}$

$1 = (4\sin y - 6\cos y)^{-1/2}\left(4\cos y\cdot\frac{dy}{dx} + 6\sin y\cdot\frac{dy}{dx}\right)$

$= \frac{dy}{dx}\left(\frac{1}{\sqrt{4\sin y - 6\cos y}}\right)(4\cos y + 6\sin y)$

$$\boxed{\frac{dy}{dx} = \frac{\sqrt{4\sin y - 6\cos y}}{4\cos y + 6\sin y}}$$

13. y = sin (cos x) + sin x * cos x

$$y' = \cos(\cos x) \cdot -\sin x + \left[\sin x \cdot -\sin x + \cos^2 x\right]$$

$$\boxed{= -\cos(\cos x)\sin x - \sin^2 x + \cos^2 x}$$

14. y = sin $^{-1}$ x^2

$$y' = \frac{1}{\sqrt{1-x^4}} \cdot 2x$$

$$\boxed{= \frac{2x}{\sqrt{1-x^4}}}$$

15. y = log$_{19}$ $\frac{x+1}{x^2+1}$

$$\cancel{y' = \frac{1}{\ln 19 \cdot \frac{x+1}{x^2+1}} \cdot (x^2 \ldots)}$$

$$y' = \log_{19}(x+1) - \log_{19}(x^2+1)$$

$$= \frac{1}{\ln 19 \cdot (x+1)} - \frac{1}{\ln 19 \, (x^2+1)} \cdot 2x$$

$$\boxed{= \frac{1}{\ln 19 \cdot (x+1)} - \frac{2x}{\ln 19 \cdot (x^2+1)}}$$

16. $y = x^{e^x}$

$\ln y = e^x \cdot \ln x$

$\frac{1}{y} \cdot \frac{dy}{dx} = e^x \cdot \frac{1}{x} + \ln x \cdot e^x$

$$\boxed{\frac{dy}{dx} = x^{e^x}\left(\frac{e^x}{x} + e^x \ln x\right)}$$

17. $y = \left(\frac{1}{b^{2x}}\right)^{2bx}$

$\ln y = 2bx \cdot \ln\left(\frac{1}{b^{2x}}\right)$

$= 2bx \cdot \ln\left(b^{-2x}\right)$

$= -4x^2 b \cdot \ln b$

$\frac{1}{y} \cdot \frac{dy}{dx} = -8xb \cdot \ln b$

$\frac{dy}{dx} = y \cdot -8x\ b \ln b$

$= \left(\frac{1}{b^{2x}}\right)^{2bx} \cdot -8x b \ln b$

18. A particle's position as it travels is given by s(t) = $t^3 - 6t^2 + 9t$. When is it speeding up?

$s'(t) = 3t^2 - 12t + 9$

$s''(t) = 6t - 12$

$$\boxed{(1,2) \cup (3, \infty)}$$