

AP Calc: HW 1.6A

2. a. $\lim_{x \rightarrow 2} [f(x) + g(x)]$
 $= \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x)$
 $= (-1) + 2$
 $= 1$

4. $\lim_{x \rightarrow -1} (x^4 - 3x)(x^2 + 5x + 3)$
 $= \lim_{x \rightarrow -1} (x^4 - 3x) \cdot \lim_{x \rightarrow -1} (x^2 + 5x + 3)$ (Product Law)
 $= (4)(-1)$ (Direct Sub)
 $= -4$

b. $\lim_{x \rightarrow 0} [f(x) - g(x)]$
 $= \lim_{x \rightarrow 0} f(x) - \lim_{x \rightarrow 0} g(x)$ ← DNE
 $= \text{DNE}$

6. $\lim_{u \rightarrow -2} \sqrt{u^4 + 3u + 6}$
 $= \lim_{u \rightarrow -2} (u^4 + 3u + 6)^{1/2}$
 $= \left[\lim_{u \rightarrow -2} (u^4 + 3u + 6) \right]^{1/2}$ (Power Law)
 $= [16]^{1/2}$ (Direct Sub)
 $= 4$

c. $\lim_{x \rightarrow -1} [f(x)g(x)]$
 $= \lim_{x \rightarrow -1} f(x) \cdot \lim_{x \rightarrow -1} g(x)$
 $= (1)(2)$
 $= 2$

j. $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)}$ ~~1/6-6661~~

$\frac{\lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} g(x)}$ ← cannot be 0
 $\lim_{t \rightarrow 2} \frac{t^2 - 2}{t^3 - 3t + 5}$
 $= \left[\lim_{t \rightarrow 2} \frac{t^2 - 2}{t^3 - 3t + 5} \right]^2$ (Power Law)
 $= \text{DNE}$

e. $\lim_{x \rightarrow 2} [x^2 f(x)]$
 $= \lim_{x \rightarrow 2} x^2 \cdot \lim_{x \rightarrow 2} f(x)$
 $= (4)(2)$
 $= 8$

f. $f(-1) + \lim_{x \rightarrow -1} g(x)$
 $= 3 + 2$
 $= 5$
 $= \left(\frac{2}{7}\right)^2$ (Direct sub)
 $= \frac{4}{49}$

$$10. a. \frac{x^2+x-6}{x-2} = x+3$$

$$\frac{(x+3)(x-2)}{x-2} = x+3$$

$$x+3; x \neq 2 = x+3$$

Not the same (domains are not the same)

b. Both functions approach the same value and will have the same graph (except for discontinuity for $\frac{x^2+x-6}{x-2}$)

$$12. \lim_{x \rightarrow -3} \frac{x^2+3x}{x^2-x-12}$$

$$= \lim_{x \rightarrow -3} \frac{x(x+3)}{(x-4)(x+3)}$$

$$= \lim_{x \rightarrow -3} \frac{x}{x-4}$$

$$= \frac{3}{7}$$

$$14. \lim_{x \rightarrow 4} \frac{x^2+3x}{x^2-x-12}$$

$$= \lim_{x \rightarrow 4} \frac{x}{x-4}$$

$$\lim_{x \rightarrow 4^+} \frac{x}{x-4}$$

$$= \frac{4^+}{4^+-4}$$

$$= \frac{4}{0^+}$$

$$= \infty$$

DNE

$$\lim_{x \rightarrow 4^-} \frac{x}{x-4}$$

$$= \frac{4^-}{4^- - 4}$$

$$= \frac{4^-}{0^-}$$

$$= -\infty$$

$$20. \lim_{t \rightarrow 1} \frac{t^4-1}{t^3-1}$$

$$= \lim_{t \rightarrow 1} \frac{(t^2+1)(t^2-1)}{(t-1)(t^2+t+1)}$$

$$= \lim_{t \rightarrow 1} \frac{(t^2+1)(t+1)(t-1)}{(t-1)(t^2+t+1)}$$

$$= \lim_{t \rightarrow 1} \frac{(t^2+1)(t+1)}{t^2+t+1}$$

$$= \frac{4}{3}$$

$$22. \lim_{u \rightarrow 2} \frac{\sqrt{4u+1}-3}{u-2}$$

$$= \lim_{u \rightarrow 2} \frac{(\sqrt{4u+1}-3)(\sqrt{4u+1}+3)}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim_{u \rightarrow 2} \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim_{u \rightarrow 2} \frac{4(u-2)}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim_{u \rightarrow 2} \frac{4}{\sqrt{4u+1}+3}$$

$$= \frac{2}{3}$$

$$16. \lim_{x \rightarrow -1} \frac{2x^2+3x+1}{x^2-2x-3}$$

$$= \lim_{x \rightarrow -1} \frac{(2x+1)(x+1)}{(x-3)(x+1)}$$

$$= \lim_{x \rightarrow -1} \frac{2x+1}{x-3}$$

$$= \frac{1}{4}$$

$$18. \lim_{h \rightarrow 0} \frac{(2+h)^3-8}{h}$$

~~$$= \lim_{h \rightarrow 0} \frac{h^3+6h^2+12h+8-8}{h}$$~~

$$= \lim_{h \rightarrow 0} \frac{h^3+6h^2+12h+8-8}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^3+6h^2+12h}{h}$$

$$= \lim_{h \rightarrow 0} h^2+6h+12$$

$$= 12$$

$$\begin{aligned}
 24. \quad \lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h} &= \lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{3 - (3+h)}{(3+h) \cdot 3}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-h}{3(h+3)} \\
 &= \lim_{h \rightarrow 0} -\frac{1}{3(h+3)}
 \end{aligned}$$

$$= -\frac{1}{9}$$

$$\begin{aligned}
 26. \quad \lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t+1} \right) &= \lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t(t+1)} \right) \\
 &= \lim_{t \rightarrow 0} \left(\frac{t+1}{t(t+1)} - \frac{1}{t(t+1)} \right) \\
 &= \lim_{t \rightarrow 0} \frac{t+1-1}{t(t+1)} \\
 &= \lim_{t \rightarrow 0} \frac{t}{t(t+1)}
 \end{aligned}$$

$$\boxed{= 1}$$

$$\begin{aligned}
 28. \quad \lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^4 - 3x^2 - 4} &= \lim_{x \rightarrow 2} \frac{(x-2)^2}{(x^2-4)(x^2+1)} \\
 &= \lim_{x \rightarrow 2} \frac{(x-2)^2}{(x-2)(x+2)(x^2+1)} \\
 &= \lim_{x \rightarrow 2} \frac{x-2}{(x+2)(x^2+1)}
 \end{aligned}$$

$$\boxed{= 0}$$

$$\begin{aligned}
 30. \quad \lim_{x \rightarrow -4} \frac{\sqrt{x^2+9} - 5}{x+4} &= \lim_{x \rightarrow -4} \frac{(\sqrt{x^2+9}-5)(\sqrt{x^2+9}+5)}{(x+4)(\sqrt{x^2+9}+5)} \\
 &= \lim_{x \rightarrow -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)} \\
 &= \lim_{x \rightarrow -4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)} \\
 &= \lim_{x \rightarrow -4} \frac{(x+4)(x-4)}{(x+4)(\sqrt{x^2+9}+5)} \\
 &= \lim_{x \rightarrow -4} \frac{x-4}{\sqrt{x^2+9}+5}
 \end{aligned}$$

$$= -\frac{8}{10}$$

$$\boxed{= -\frac{4}{5}}$$

$$\begin{aligned}
 32. \quad \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} &= \lim_{h \rightarrow 0} \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 - (x+h)^2}{x^2(x+h)^2 h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2 h}
 \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 - x^2 - 2xh - h^2}{x^2(x+h)^2 h}$$

$$= \lim_{h \rightarrow 0} \frac{-2xh - h^2}{x^2(x+h)^2 h}$$

$$= \lim_{h \rightarrow 0} \frac{-2x - h}{x^2(x+h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{-2x - h}{x^2(x+h)^2}$$

$$= -\frac{2x}{x^4}$$

$$\boxed{= -\frac{2}{x^3}}$$