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AP Calc AB: 1.6B

38. Given that:
$2x \le g(x) \le x^4 - x^2 + 2$

$\lim_{x\to 1} 2x = 2$

$\lim_{x\to 1} [x^4 - x^2 + 2] = 2$

Because the two limits above are equal, by squeeze thrm., we can conclude that $\boxed{\lim_{x\to 1} g(x) = 2}$

42. $\lim_{x\to -6} \frac{2x+12}{|x+6|}$

$\lim_{x\to -6^-} \frac{2x+12}{|x+6|}$ $\qquad$ $\lim_{x\to -6^+} \frac{2x+12}{|x+6|}$

$= \lim_{x\to -6^-} \frac{2x+12}{-(x+6)}$ $\qquad$ $= \lim_{x\to -6^+} \frac{2x+12}{x+6}$

$= \lim_{x\to -6^-} \frac{2(x+6)}{-(x+6)}$ $\qquad$ $= \lim_{x\to -6^+} \frac{2(x+6)}{x+6}$

$= -2$ $\qquad$ $= 2$

$\boxed{\text{Limit DNE because one-sided limits are not equal}}$

40. $\lim_{x\to 0^+} \sqrt{x}\left[1 + \sin^2\left(\frac{2\pi}{x}\right)\right] = 0$

use sqz. thrm to determine/prove

$-1 \le \sin\left(\frac{2\pi}{x}\right) \le 1$

$0 \le \sin^2\left(\frac{2\pi}{x}\right) \le 1$

$1 \le 1 + \sin^2\left(\frac{2\pi}{x}\right) \le 2$

$\sqrt{x} \le \sqrt{x}\left[1 + \sin^2\left(\frac{2\pi}{x}\right)\right] \le 2\sqrt{x}$

$\lim_{x\to 0^+} \sqrt{x} = 0$

$\lim_{x\to 0^+} 2\sqrt{x} = 0$

$\therefore$ By squeeze thrm

$\boxed{\lim_{x\to 0^+} \sqrt{x}\left[1 + \sin^2\left(\frac{2\pi}{x}\right)\right] = 0}$

44. $\lim_{x\to -2} \frac{2-|x|}{2+x}$

$\lim_{x\to -2^-} \frac{2-|x|}{2+x}$ $\qquad$ $\lim_{x\to -2^+} \frac{2-|x|}{2+x}$

$= \lim_{x\to -2^-} \frac{2-(-x)}{2+x}$ $\qquad$ $\lim_{x\to -2^+} \frac{2-(-x)}{2+x}$

$= \lim_{x\to -2^-} \frac{2+x}{2+x}$ $\qquad$ $\lim_{x\to -2^+} \frac{2+x}{2+x}$

$= 1$ $\qquad$ $= 1$

$\boxed{\lim_{x\to -2} \frac{2-|x|}{2+x} = 1}$

46. $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

$= \lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x}\right)$

$= \lim_{x\to 0^+} (0)$

$\boxed{= 0}$

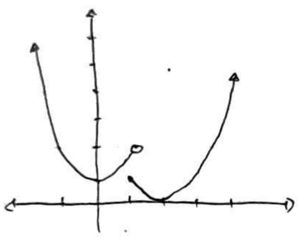
50. a. $\lim_{x\to 1^-} f(x)$ $\qquad$ $\lim_{x\to 1^+} f(x)$

$= \lim_{x\to 1^-} (x^2+1)$ $\qquad$ $= \lim_{x\to 1^+} (x-2)^2$

$\boxed{=2}$ $\qquad$ $\boxed{=1}$

b. No.

c.

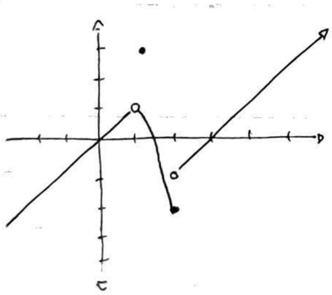


56. The left hand limit is necessary as if it weren't, you'd have to $\sqrt{1-1}$, or 0. This wouldn't make sense as an equation.

Also, no object in physics is possible (at least so far) to travel at the speed of light, or c.

52. a. i. $\boxed{1}$, ii. $\boxed{1}$, iii. $\boxed{3}$, iv. $\boxed{-2}$, v. $\boxed{-1}$, vi. $\boxed{DNE}$

b.



60. a. $\lim_{x\to 0} \frac{f(x)}{x^2} = 5$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \cdot \lim_{x\to 0} \frac{f(x)}{x^2}$

$= 0 \cdot 5$

$\boxed{=0}$

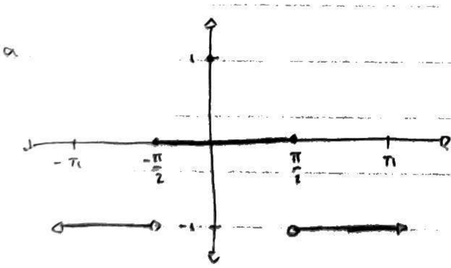
b. $\lim_{x\to 0} \frac{f(x)}{x^2} = 5$

$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} x \cdot \lim_{x\to 0} \frac{f(x)}{x^2}$

$= 0 \cdot 5$

$\boxed{=0}$

54. a



b. i. $\boxed{0}$ ii. $\boxed{0}$ iii. $\boxed{-1}$ iv. $\boxed{DNE}$

c. $\frac{\pi}{2} + 2\pi k$, $\frac{3\pi}{2} + 2\pi k$

$\boxed{\mathbb{R}, \text{except } \frac{\pi}{2} + \pi k}$

61. Because there are an infinite number of rational & irration numbers, we use squeeze thm:

min value

max possible value

$0 \leq f(x) \leq x^2$

$\lim_{x\to 0} 0 \leq \lim_{x\to 0} f(x) \leq \lim_{x\to 0} x^2$

$\lim_{x\to 0} 0 = 0$

$\lim_{x\to 0} x^2 = 0$

$\therefore$ By squeeze, $\boxed{\lim_{x\to 0} f(x) = 0}$

64. $\lim_{x\to 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1}$

$= \lim_{x\to 2} \frac{(\sqrt{6-x} - 2)(\sqrt{6-x} + 2)(\sqrt{3-x} + 1)}{(\sqrt{3-x} - 1)(\sqrt{3-x} + 1)(\sqrt{6-x} + 2)}$

$= \lim_{x\to 2} \frac{((6-x) - 4)(\sqrt{3-x} + 1)}{((3-x) - 1)(\sqrt{6-x} + 2)}$

$= \lim_{x\to 2} \frac{(-x+2)(\sqrt{3-x} + 1)}{(-x+2)(\sqrt{6-x} + 2)}$

$= \lim_{x\to 2} \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2}$

$= \frac{2}{4}$

$\boxed{= \frac{1}{2}}$