James Ding

AP Calc AB: 1.7A

16. Prove: $\lim_{x\to 4}(2x-5)=3$

Scratch:

If $0<|x-4|<\delta$, then $|(2x-5)-3|<\varepsilon$

$|(2x-5)-3|<\varepsilon$

$|2x-8|<\varepsilon$

$|2||x-4|<\varepsilon$

$2|x-4|<\varepsilon$

$|x-4|<\frac{\varepsilon}{2}$

Proof:

Given $\varepsilon>0$, pick $\delta=\frac{\varepsilon}{2}$ if $0<|x-4|<\delta$,

then $|(2x-5)-3|<\varepsilon$

$|(2x-5)-3|$ $\cancel{<\varepsilon}$

$2|x-4|$ $\cancel{<\varepsilon}$

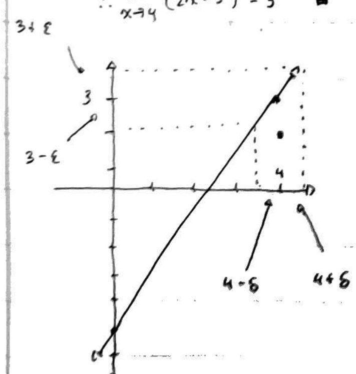
$2|x-4|<2\delta$

$2|x-4|<2\left(\frac{\varepsilon}{2}\right)$

$2|x-4|<\varepsilon$

Thus $|(2x-5)-3|<\varepsilon$

$\therefore \lim_{x\to 4}(2x-5)=3$ ∎



18. Prove: $\lim_{x\to -2}(3x+5)=-1$

Scratch:

If $0<|x+2|<\delta$, then $|(3x+5)+1|<\varepsilon$

$|(3x+5)+1|<\varepsilon$

$|3x+6|<\varepsilon$

$|3||x+2|<\varepsilon$

$3|x+2|<\varepsilon$

$|x+2|<\frac{\varepsilon}{3}$

Proof:

Given $\varepsilon>0$, pick $\delta=\frac{\varepsilon}{3}$ if $0<|x+2|<\delta$,

then $|(3x+5)+1|<\varepsilon$

$|(3x+5)+1|$ $\cancel{<\varepsilon}$

[illegible]

$3|x+2|$ $\cancel{<\varepsilon}$

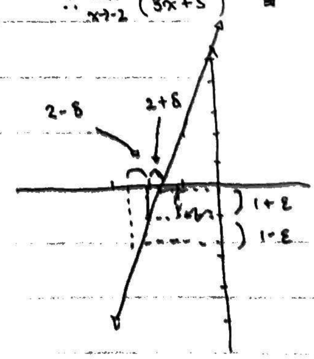
$3|x+2|<3\delta$

$3|x+2|<3\left(\frac{\varepsilon}{3}\right)$

$3|x+2|<\varepsilon$

Thus $|(3x+5)+1|<\varepsilon$

$\therefore \lim_{x\to -2}(3x+5)$ ∎



(in the graph, segments don't look equal, but are supposed to be equal my drawing skills suck)

20. Prove: $\lim_{x \to 10} \left(3 - \frac{4}{5}x\right) = -5$

Scratch:

If $0 < |x - 10| < \delta$, then $|(3 - \frac{4}{5}x) + 5| < \varepsilon$

$|(3 - \frac{4}{5}x) + 5| < \varepsilon$

$|-\frac{4}{5}x + 8| < \varepsilon$

$|-1||\frac{4}{5}x - 8| < \varepsilon$

$|\frac{5}{4}||\frac{4}{5}x - 8| < |\frac{5}{4}|\varepsilon$

$|x - 10| < \frac{5\varepsilon}{4}$

Proof:

Given $\varepsilon > 0$, pick $\delta = \frac{5\varepsilon}{4}$ if $0 < |x - 10| < \delta$, then $|(3 - \frac{4}{5}x) + 5| < \varepsilon$

$|(3 - \frac{4}{5}x) + 5| \cancel{< \varepsilon}$

$|-\frac{4}{5}x + 8| \cancel{< \varepsilon}$

$|\frac{4}{5}x - 8| \cancel{< \varepsilon}$

$\frac{4}{5}|x - 10| \cancel{< \varepsilon}$

$\frac{4}{5}|x - 10| < \frac{4}{5}\delta$

$\frac{4}{5}|x - 10| < \frac{4}{5}\left(\frac{5\varepsilon}{4}\right)$

$\frac{4}{5}|x - 10| < \varepsilon$

Thus $|(3 - \frac{4}{5}x) + 5| < \varepsilon$

$\therefore \lim_{x \to 10} \left(3 - \frac{4}{5}x\right) = -5$ ∎

22. Prove: $\lim_{x \to -1.5} \frac{9-4x^2}{3+2x} = 6$

Scratch:

If $0 < |x + \frac{3}{2}| < \delta$, then $\left| \frac{9-4x^2}{3+2x} - 6 \right| < \varepsilon$

$\left| \frac{9-4x^2}{3+2x} - 6 \right| < \varepsilon$

$\left| \frac{-4x^2+9}{3+2x} - 6 \right| < \varepsilon$

$\left| \frac{-(4x^2-9)}{3+2x} - 6 \right| < \varepsilon$

$\left| \frac{-(2x+3)(2x-3)}{(3+2x)} - 6 \right| < \varepsilon$

$| -(2x-3) - 6 | < \varepsilon$

$| -2x + 3 - 6 | < \varepsilon$

$| -2x - 3 | < \varepsilon$

$|-2| |x + \frac{3}{2}| < \varepsilon$

$2 |x + \frac{3}{2}| < \varepsilon$

$|x + \frac{3}{2}| < \frac{\varepsilon}{2}$

Proof:

Given $\varepsilon > 0$, pick $\delta = \frac{\varepsilon}{2}$ if $0 < |x + \frac{3}{2}| < \delta$, then

$\left| \frac{9-4x^2}{3+2x} - 6 \right| < \varepsilon$

$\left| \frac{9-4x^2}{3+2x} - 6 \right|$

$| -2x - 3 |$

$2 |x + \frac{3}{2}| < 2\delta$

$2 |x + \frac{3}{2}| < 2\left(\frac{\varepsilon}{2}\right)$

$2 |x + \frac{3}{2}| < \varepsilon$

Thus $\left| \frac{9-4x^2}{3+2x} - 6 \right| < \varepsilon$

$\therefore \lim_{x \to -1.5} \frac{9-4x^2}{3+2x} = 6$ ∎

24. Prove: $\lim_{x \to a} c = c$

If $0 < |x - a| < \delta$, then $|c - c| < \varepsilon$

$|c - c| < \varepsilon$

$0 < \varepsilon$

Proof:

Given $\varepsilon > 0$, let $\delta > 0$ if $0 < |x - a| \delta$, then $|c - c| < \varepsilon$

$0 < \delta$

Thus, $\lim_{x \to a} c = c$ ∎

25. Prove: $\lim_{x \to 0} x^2 = 0$

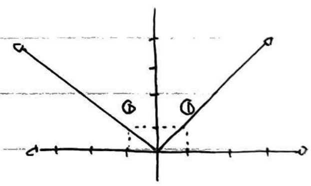
Scratch:

If $0 < |x - 0| < \delta$, then $|x^2 - 0| < \varepsilon$

$|x^2| < \varepsilon$

$|x||x| < \varepsilon$

$|x||x| < M|x| < \varepsilon$



Assume:

$\delta \leq 1$

$\delta < \varepsilon$

$M = 1$

Proof

Given $\varepsilon > 0$, pick

$\delta = \min(1, \varepsilon)$. If $0 < |x| < \delta$, then

consider $|x^2| < \varepsilon$

$|x^2| < |x| \delta$ $\qquad |x| < 1$

$|x^2| < (1)(\varepsilon)$ $\qquad -1 < x < 1$

Thus $|x^2| < \varepsilon$ ∎ $\qquad \delta \leq \varepsilon$

$\therefore \lim_{x \to 0} x^2 = 0$