James Diz

AP calc AB: 1.7B

26. Prove: $\lim_{x \to 0} x^3 = 0$

Scratch:

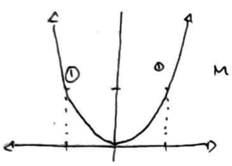
If $0 < |x-0| < \delta$, then $|x^3 - 0| < \varepsilon$

$|x^3 - 0| < \varepsilon$

$|x^3| < \varepsilon$

$|x||x^2| < \varepsilon$

$|x||x^2| < M|x| < \varepsilon$



$M = 1$

Assume:

$\delta \leq 1$

$\delta < \varepsilon$

Proof:

Given $\varepsilon > 0$, pick $\delta = \min(1, \varepsilon)$

If $0 < |x-0| < \delta$, then consider $|x^3 - 0| < \varepsilon$

$|x||x^2| < |x^2|\delta$

$|x^2|\delta < (1)\delta = \varepsilon$

$|x| < 1$

$-1 < x < 1$

$0 < \underline{x^2 < 1}$

$\delta < \varepsilon$

Thus, $|x^3 - 0| < \varepsilon$

$\therefore \lim_{x \to 0} x^3 = 0$ ■

28. Prove: $\lim_{x \to -6^+} \sqrt[8]{6+x} = 0$

Scratch:

If $-6 < x < -6 + \delta$, then $|\sqrt[8]{6+x} - 0| < \varepsilon$

$|\sqrt[8]{6+x} - 0| < \varepsilon$ ----> $0 < x + 6 < \delta$

$|\sqrt[8]{6+x}| < \varepsilon$

$|6+x| < \varepsilon^8$

Proof:

Given $\varepsilon > 0$, pick $\delta = \varepsilon^8$. If $-6 < x < -6 + \delta$, then consider $|\sqrt[8]{6+x} - 0| < \varepsilon$

$|\sqrt[8]{6+x} - 0|$

$|\sqrt[8]{6+x}| < \sqrt[8]{\delta}$

$|\sqrt[8]{6+x}| < \sqrt[8]{\varepsilon^8}$

Thus, $|\sqrt[8]{6+x} - 0| < \varepsilon$

$\therefore \lim_{x \to -6^+} \sqrt[8]{6+x} = 0$ ■

30. Prove: $\lim_{x \to 2} (x^2+2x-7) = 1$

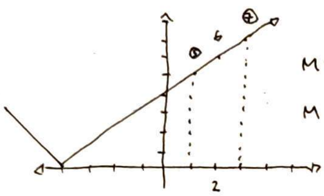
Scratch:

If $0<|x-2|<\delta$, then $|(x^2+2x-7)-1|<\varepsilon$

$|x^2+2x-8|<\varepsilon$

$|x-2||x+4|<\varepsilon$

$|x-2||x+4| < M|x-2| < \varepsilon$



$M=7$

$M|x-2|<\varepsilon$

$|x-2| < \frac{\varepsilon}{7}$

Assume:

$\delta \le 1$

$\delta < \frac{\varepsilon}{7}$

Proof:

Given $\varepsilon>0$, pick $\delta = \min(1, \frac{\varepsilon}{7})$. If $0<|x-2|<\delta$ then consider $|(x^2+2x-7)-1|<\varepsilon$

$|(x^2+2x-7)-1|$

$|x-2||x+4| < |x+4|\delta$

$|x+4|\delta < 7\delta = 7\left(\frac{\varepsilon}{7}\right) = \varepsilon$

$|x-2|<1$

$-1<x-2<1$

$1<x<3$

$5<x<7$

$\delta \le \frac{\varepsilon}{7}$

Thus $|(x^2+2x-7)-1|<\varepsilon$

$\therefore \lim_{x \to 2} (x^2+2x-7) = 1$ ∎

29. Prove: $\lim_{x \to 2} (x^2-4x+5) = 1$

Scratch:

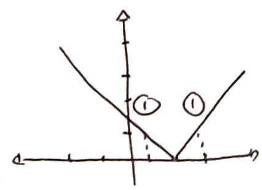
If $0<|x-2|<\delta$, then $|(x^2-4x+5)-1|<\varepsilon$

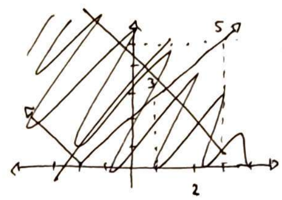
$|(x^2-4x+5)-1|<\varepsilon$

$|x^2-4x+4|<\varepsilon$

$|x-2||x-2|<\varepsilon$

$|x-2||x-2| < M|x-2| < \varepsilon$





$M=1$

Assume:

$\delta \le 1$

$\delta < \frac{\varepsilon}{5}$

$M|x-2|<\varepsilon$; $|x-2| < \varepsilon$

Proof:

Given that $\varepsilon>0$, pick $\delta = \min(1, \varepsilon)$. If $0<|x-2|<\delta$ then consider $|(x^2-4x+5)-1|<\varepsilon$

$|(x^2-4x+5)-1|$

$|x-2||x-2|$

$|x-2||x-2| < |x-2|\delta$

$|x-2|\delta < 1\delta = \varepsilon$

$|x-2|<1$

$-1<x-2<1$

Thus, $|(x^2-4x+5)-1|<\varepsilon$

$\therefore \lim_{x \to 2} (x^2-4x+5) = 1$ ∎

32. Prove: $\lim_{x\to 2} x^3 = 8$

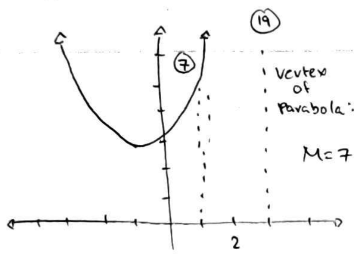
Scratch:

If $0 < |x-2| < \delta$, then $|x^3 - 8| < \varepsilon$

$|x^3 - 8| < \varepsilon$

$|x-2||x^2+2x+4| < \varepsilon$

$|x-2||x^2+2x+4| < M|x-2| < \varepsilon$



Vertex of Parabola: $(-1, 3)$

$M = 7$

Assume:

$\delta \le 1$

$\delta < \frac{\varepsilon}{19}$

$19|x-2| < \varepsilon$; $|x-2| < \frac{\varepsilon}{19}$

Proof:

Given $\varepsilon > 0$, pick $\delta = \min\left(1, \frac{\varepsilon}{19}\right)$. If $0<|x-2|<\delta$

then consider $|x^3 - 8| < \varepsilon$

$|x^3 - 8|$

$|x-2||x^2+2x+4|$

$|x-2||x^2+2x+4| < |x^2+2x+4|\delta$

$|x^2+2x+4|\delta < 19\delta = 19\left(\frac{\varepsilon}{19}\right) = \varepsilon$

Thus, $|x^3 - 8| < \varepsilon$

$\therefore \lim_{x\to 2} x^3 = 8$

$|x-2| < 1$

$-1 < x-2 < 1$

$1 < x < 3$

$1 < x^2 < 9$

~~[illegible]~~

$3 < x^2 + 2x < 15$

$7 < x^2+2x+4 < 19$

$\delta < \frac{\varepsilon}{19}$

41. $\frac{1}{(x+3)^4} > 10{,}000$

$(x+3)^4 < \frac{1}{10{,}000}$

$|x+3|^4 < \frac{1}{10{,}000}$

$|x+3| < \frac{1}{\sqrt[4]{10{,}000}} = \frac{1}{10}$

$\boxed{|x+3| < \frac{1}{10}}$