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AP Calc AB: HW 1.8B

26. $G(x) = \frac{x^2+1}{2x^2-x-1}$

$G(x)$ is undefined when $2x^2 - x - 1 = 0$

$2x^2 - x - 1 = 0$
$(2x+1)(x-1) = 0$
$x = -\frac{1}{2}, 1$

$\boxed{x \in (-\infty, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, 1) \cup (1, \infty)}$

$\therefore$ Because $G(x)$ is a rational function, it must be continuous at every number in it's domain

28. $h(x) = \frac{\sin x}{x+1}$

$h(x)$ is undefined when $x+1=0$

$x+1=0$
$x=-1$

$\boxed{x \in (-\infty, -1) \cup (-1, \infty)}$

$\therefore$ Because $\sin x$ is a trigonometric function, it is continuous everywhere in its domain $(-\infty, \infty)$. $x+1$ is also a continuous function everywhere because it is linear. Because $h(x)$ follows the form $\frac{f(x)}{g(x)}$, it is continuous everywhere in its domain, as $f(x) = \sin x$ and $g(x) = x+1$

32. $F(x) = \sin(\cos(\sin x))$

- $\sin x$ is continuous everywhere
- $\cos x$ is continuous everywhere, which means ~~[illegible]~~ $\cos(\sin x)$ must be continuous everywhere
- Because $\sin x$ is continuous everywhere, $\sin(\cos(\sin x))$ must be continuous everywhere

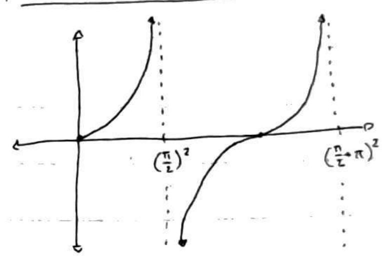
$\boxed{x \in \mathbb{R}}$

$\therefore$ $F(x)$ must be continuous everywhere

34. $y = \tan\sqrt{x}$

The graph of $\tan x$ is discontinuous at $\frac{\pi}{2} + \pi k$

$\boxed{\therefore \text{ The graph of } \tan\sqrt{x} \text{ is discontinuous at } \left(\tfrac{\pi}{2} + \pi k\right)^2 \text{ and when } x < 0}$



36. $\lim_{x \to \pi} \sin(x + \sin x)$

$x + \sin x$ is continuous everywhere as x is continuous everywhere, along with $\sin x$ being continuous. This means $x + \sin x$ follows the form $f(x) + g(x)$, which mean it too must be continuous everywhere.

Because $x + \sin x$ is continuous everywhere, $\sin(x + \sin x)$ must also be continuous everywhere

$\therefore \lim_{x \to \pi} \sin(x + \sin x) = \sin(\pi + \sin \pi)$

$= \sin(\pi + 0)$

$\boxed{= 0}$

40. $f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$

Because $f(x)$ is $\sin x$ when $x < \pi/4$

$f(x)$ must be continuous when $x < \pi/4$

because $\sin x$ is continuous everywhere.

Because $f(x)$ is $\cos x$ when $x \geq \pi/4$

$f(x)$ must be continuous when $x \geq \pi/4$

because $\cos x$ is continuous everywhere

$\therefore$ Because $f(x)$ is continuous when $x < \pi/4$ and when $x > \pi/4$, $f(x)$ is continuous everywhere if $\sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4})$, which is true. Therefore, $f(x)$ is continuous on the interval $(-\infty, \infty)$

45. $f(x) = \begin{cases} cx^2 + 2x \; ; & x < 2 \\ x^3 - cx \; ; & x \geq 2 \end{cases}$

both peicewise parts are cont everywhere

To make $f(x)$ continuous, both parts of the peicewise function must be equal when $x = 2$

$cx^2 + 2x = x^3 - cx$

$c(2)^2 + 2(2) = (2)^3 - c(2)$

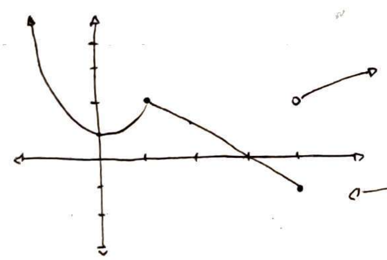
$4c + 4 = 8 - 2c$

$6c = 4$

$c = \frac{4}{6} = \frac{2}{3}$

$\boxed{c = \frac{2}{3}}$

42.



discontinuous at 4

Discontinuous at 4, right

Plus:

30. $B(x) = \dfrac{\tan x}{\sqrt{4-x^2}}$

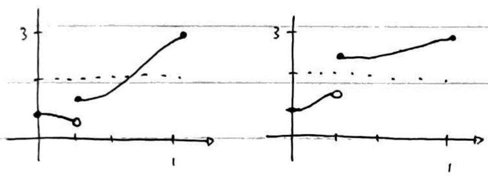
Because $\tan x$ is a trigonometric function it is continuous everywhere in its domain.

Because $4-x^2$ is a polynomial function, it is continuous everywhere. Therefore $\sqrt{4-x^2}$ must also be continuous everywhere as it follows the form $f(g(x))$.

Thus $B(x)$ is continuous everywhere in its domain as it follows the form $\dfrac{f(x)}{g(x)}$

$$\boxed{x \in \left(-2, -\tfrac{\pi}{2}\right) \cup \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right) \cup \left(\tfrac{\pi}{2}, 2\right)}$$

50.



54. $f(x) = x - \sqrt{x} - \frac{2}{x}$; Prove a root between $(2,3)$

$f(x)$ is continuous everywhere in it's domain, ~~[illegible]~~ therefore it is continuous on the interval $[2,3]$

$f(2) = 2 - \sqrt{2} - \frac{2}{2} = 1 - \sqrt{2}$ (Negative)

$f(3) = 3 - \sqrt{3} - \frac{2}{3}$ (Positive)

$\therefore$ By IVT, there exists ~~[illegible]~~ root (s) on the interval $(2,3)$ ■

56. $f(x) = x^2 - x - \sin x$; Prove a root between $(1,2)$

$f(x)$ is a continuous function everywhere in it's domain, therefore it is continuous on the interval $[1,2]$

~~$f(1) = 1 - 1 - \sin(1)$ (Negative)~~

$f(2) = 4 - 2 - \sin(2)$ (Positive)

$\therefore$ By IVT, there exists root(s) on the interval $(1,2)$ ■