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AP Calc AB: HW 2.1A

3. a. i. $m = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to 1} \frac{4x-x^2-3}{x-1}$

$= \lim_{x\to 1} \frac{-(x^2-4x+3)}{x-1}$

$= \lim_{x\to 1} \frac{-(x-1)(x-3)}{x-1}$

$= \lim_{x\to 1} -x+3$

$\boxed{=2}$

ii. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$= \lim_{h\to 0} \frac{4(h+1)-(h+1)^2-3}{h}$

$= \lim_{h\to 0} \frac{4h+4-h^2-2h-1-3}{h}$

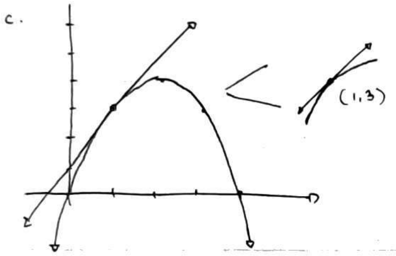
$= \lim_{h\to 0} \frac{h^2+2h}{h}$

$= \lim_{h\to 0} h+2$

$\boxed{=2}$

b. $y-3 = 2(x-1)$

c.



6. $m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$= \lim_{h\to 0} \frac{(h+2)^3-3(h+2)+1-3}{h}$

$= \lim_{h\to 0} \frac{h^3+6h^2+12h+8-3h-6-2}{h}$

$= \lim_{h\to 0} \frac{h^3+6h^2+9h}{h}$

$= \lim_{h\to 0} h^2+6h+9$

$= 9$

$\boxed{y-3 = 9(x-2)}$

7. $m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$= \lim_{h\to 0} \frac{\sqrt{h+1}-1}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{h+1}-1)(\sqrt{h+1}+1)}{h(\sqrt{h+1}+1)}$

$= \lim_{h\to 0} \frac{h+1-1}{h(\sqrt{h+1}+1)}$

$= \lim_{h\to 0} \frac{1}{\sqrt{h+1}+1}$

$= \frac{1}{2}$

$\boxed{y-1 = \frac{1}{2}(x-1)}$

10a. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$

$= \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$

$= \lim_{h\to 0} \frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$

$= \lim_{h\to 0} \frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$

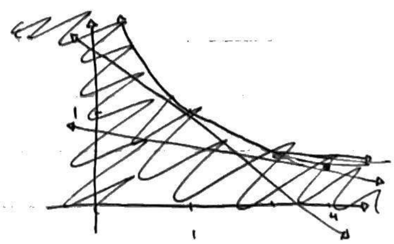
$= \lim_{h\to 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$

$= \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$

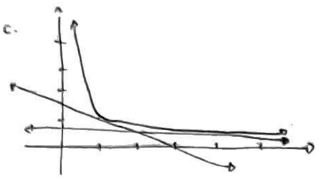
$= -\frac{1}{2x\sqrt{x}}$

b. At $(1,1)$: $m = \frac{-1}{2x\sqrt{x}} = -\frac{1}{2}$ At $(4,\frac{1}{2})$: $m = \frac{-1}{2x\sqrt{x}} = -\frac{1}{16}$

$y-1 = -\frac{1}{2}(x-1)$ $y-\frac{1}{2} = -\frac{1}{16}(x-4)$



c.



14. a. $\lim_{h\to 0} \dfrac{10(h+1) - 1.86(h+1)^2 - 8.14}{h}$

$= \lim_{h\to 0} \dfrac{10h + 10 - 1.86h^2 - 3.72h - 1.86 - 8.14}{h}$

$= \lim_{h\to 0} \dfrac{-1.86h^2 + 6.28h}{h}$

$= \lim_{h\to 0} -1.86h + 6.28$

$\boxed{= 6.28 \text{ m/s}}$

b. $\lim_{h\to 0} \dfrac{10(h+x) - 1.86(h+x)^2 - (10x - 1.86x^2)}{h}$

$= \lim_{h\to 0} \dfrac{10h + 10x - 1.86h^2 - 3.72hx - 1.86x^2 - 10x + 1.86x^2}{h}$

$= \lim_{h\to 0} \dfrac{10h - 1.86h^2 - 3.72hx}{h}$

$= \lim_{h\to 0} 10 - 1.86h - 3.72x$

$\boxed{= 10 - 3.72x}$

c. $0 = 10t - 1.86t^2$

$= t(10 - 1.86t)$

$0 = 10 - 1.86t$

$t = 0, \dfrac{10}{1.86}$

cannot be zero

$\boxed{t = \dfrac{10}{1.86} \approx 5.376 \text{ s}}$

d. $10 - 3.72\left(\dfrac{10}{1.86}\right) = \boxed{-10 \text{ m/s}}$

16. a. i. $[4, 8]$

$\dfrac{f(8) - f(4)}{8 - 4} = \boxed{0}$

ii. $[6, 8]$

$\dfrac{f(8) - f(6)}{8 - 6} = \boxed{1}$

iii. $[8, 10]$

$\dfrac{f(10) - f(8)}{10 - 8} = \boxed{3}$

iv. $[8, 12]$

$\dfrac{f(12) - f(8)}{12 - 8} = \boxed{4}$

b. $v = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

$= \lim_{h\to 0} \dfrac{\frac{1}{2}(h+8)^2 - 6(h+8) + 23 - 7}{h}$

$= \lim_{h\to 0} \dfrac{\frac{1}{2}(h^2 + 16h + 64) - 6h \cancel{-48} - 48 + 23 - 7}{h}$

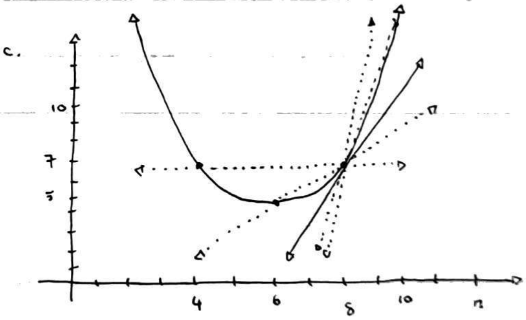
$= \lim_{h\to 0} \dfrac{\frac{1}{2}h^2 + 8h + 32 - 6h - 32}{h}$

$= \lim_{h\to 0} \dfrac{\frac{1}{2}h^2 + 2h}{h}$

$= \lim_{h\to 0} \frac{1}{2}h + 2$

$\boxed{= 2 \text{ fps}}$

c.



22. $\boxed{f(4) = 3}$

$f'(4) = \frac{3-2}{4-0} = \boxed{\frac{1}{4}}$

28. $g'(1) = \lim_{h \to 0} \frac{f(h+1) - f(1)}{h}$

$= \lim_{h \to 0} \frac{(h+1)^4 - 2 + 1}{h}$

$= \lim_{h \to 0} \frac{h^4 + 4h^3 + 6h^2 + 4h + 1 - 1}{h}$

$= \lim_{h \to 0} \frac{h^4 + 4h^3 + 6h^2 + 4h}{h}$

$= \lim_{h \to 0} h^3 + 4h^2 + 6h + 4$

$= 4$

$\boxed{y + 1 = 4(x-1)}$

34. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

$= \lim_{h \to 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2}$

$= \lim_{h \to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2}$

$= \lim_{h \to 0} \frac{x^2 - x^2 - 2xh - h^2}{hx^2(x+h)^2}$

$= \lim_{h \to 0} \frac{-2xh - h^2}{hx^2(x+h)^2}$

$= \lim_{h \to 0} \frac{-2x - h}{x^2(x+h)^2}$

$= \frac{-2x}{x^2(x)^2}$

$\boxed{= -\frac{2}{x^3}}$

36. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{4}{\sqrt{1-(x+h)}} - \frac{4}{\sqrt{1-x}}}{h}$

$= \lim_{h \to 0} \frac{4\sqrt{1-x} - 4\sqrt{1-x-h}}{\sqrt{1-x-h}\, h \sqrt{1-x}}$

$= \lim_{h \to 0} \frac{(4\sqrt{1-x} - 4\sqrt{1-x-h})(4\sqrt{1-x} + 4\sqrt{1-x-h})}{h\sqrt{1-x}\sqrt{1-x-h}\,(4\sqrt{1-x} + 4\sqrt{1-x-h})}$

$= \lim_{h \to 0} \frac{16(1-x) - 16(1-x-h)}{h\sqrt{1-x}\sqrt{1-x-h}\,(4\sqrt{1-x} + 4\sqrt{1-x-h})}$

$= \lim_{h \to 0} \frac{16 - 16x - 16 + 16x + 16h}{h\sqrt{1-x}\sqrt{1-x-h}\,(4\sqrt{1-x} + 4\sqrt{1-x-h})}$

$= \lim_{h \to 0} \frac{16h}{h\sqrt{1-x}\sqrt{1-x-h}\,(4\sqrt{1-x} + 4\sqrt{1-x-h})}$

$= \lim_{h \to 0} \frac{16}{\sqrt{1-x}\sqrt{1-x-h}\,(4\sqrt{1-x} + 4\sqrt{1-x-h})}$

$= \frac{16}{(1-x)(8\sqrt{1-x})}$

$\boxed{= \frac{2}{(1-x)\sqrt{1-x}}}$