James Ding

AP Calc AB: HW 2.3B

26. $B(u) = (u^3+1)(2u^2-4u-1)$

$B'(u) = (u^3+1)\frac{\partial}{\partial u}(2u^2-4u-1) + (2u^2-4u-1)\frac{\partial}{\partial u}(u^3+1)$

$= (u^3+1)(4u-4) + (2u^2-4u-1)(3u^2)$

28. $J(v) = (v^3-2v)(v^{-4}+v^{-2})$

$J'(v) = (v^3-2v)\frac{\partial}{\partial v}(v^{-4}+v^{-2}) + (v^{-4}+v^{-2})\frac{\partial}{\partial v}(v^3-2v)$

$= (v^3-2v)(-4v^{-5}-2v^{-3}) + (v^{-4}+v^{-2})(3v^2-2)$

30. $h(t) = \frac{6t+1}{6t-1}$

$h'(t) = \frac{(6t-1)\frac{\partial}{\partial t}(6t+1) - (6t+1)\frac{\partial}{\partial t}(6t-1)}{(6t-1)^2}$

$= \frac{(6t-1)(6) - (6t+1)(6)}{(6t-1)^2}$

$= \frac{36t-6-(36t+6)}{(6t-1)^2}$

$= -\frac{12}{(6t-1)^2}$

32. $y = \frac{1}{t^3+2t^2-1}$

$y' = \frac{(t^3+2t^2-1)\frac{\partial}{\partial t}1 - 1\cdot\frac{\partial}{\partial t}(t^3+2t^2-1)}{(t^3+2t^2-1)^2}$

$= \frac{-(3t^2+4t)}{(t^3+2t^2-1)^2}$

$= \frac{-3t^2-4t}{(t^3+2t^2-1)^2}$

34. $y = \frac{(u+2)^2}{1-u}$

$= \frac{u^2+4u+4}{1-u}$

$y' = \frac{(1-u)\frac{\partial}{\partial u}(u^2+4u+4) - (u^2+4u+4)\frac{\partial}{\partial u}(1-u)}{(1-u)^2}$

$= \frac{(1-u)(2u+4) - (u^2+4u+4)(-1)}{(1-u)^2}$

$= \frac{(1-u)(2u+4) + (u^2+4u+4)}{(1-u)^2}$

36. $y = \frac{\sqrt{x}}{2+x}$

$y' = \frac{(2+x)\frac{\partial}{\partial x}\sqrt{x} - \sqrt{x}\frac{\partial}{\partial x}(2+x)}{(2+x)^2}$

$= \frac{(2+x)(\frac{1}{2}x^{-1/2}) - \sqrt{x}}{(x+2)^2}$

38. $y = \frac{cx}{1+cx}$

$y' = \frac{(1+cx)\frac{\partial}{\partial x}cx - (cx)\frac{\partial}{\partial x}(1+cx)}{(1+cx)^2}$

$= \frac{(1+cx)c - cx\cdot c}{(1+cx)^2}$

$= \frac{(1+cx)c - cx\cdot c}{(1+cx)^2}$

$= \frac{c+c^2x-c^2x}{(1+cx)^2}$

$= \frac{c}{(1+cx)^2}$

40. $A(v) = v^{2/3}(2v^2 + 1 - v^{-2})$

$A'(v) = v^{2/3}\frac{\partial}{\partial v}(2v^2 + 1 - v^{-2}) + (2v^2 + 1 - v^{-2})\frac{\partial}{\partial v} v^{2/3}$

$= v^{2/3}(4v + 2v^{-3}) + (2v^2 + 1 - v^{-2})\left(\frac{2}{3}v^{-1/3}\right)$

42. $F(t) = \frac{At}{Bt^2 + Ct^3}$

$F'(t) = \frac{(Bt^2 + Ct^3)\frac{\partial}{\partial t}At - At\frac{\partial}{\partial t}(Bt^2 + Ct^3)}{(Bt^2 + Ct^3)^2}$

$= \frac{(Bt^2 + Ct^3)(A) - At(2Bt + 3Ct^2)}{(Bt^2 + Ct^3)^2}$

44. $f(x) = \frac{ax + b}{cx + d}$

$f'(x) = \frac{(cx+d)\frac{\partial}{\partial x}(ax+b) - (ax+b)\frac{\partial}{\partial x}(cx+d)}{(cx+d)^2}$

$= \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2}$

$= \frac{acx + da - acx - bc}{(cx+d)^2}$

$= \frac{da - bc}{(cx+d)^2}$

52. $y = 2x^3 - x^2 + 2$

$y' = 6x^2 - 2x$

$6(1)^2 - 2(1) = 4$

$y - 3 = 4(x - 1)$

58. $y = \frac{\sqrt{x}}{x+1}$

$y' = \frac{(x+1)\frac{\partial}{\partial x}\sqrt{x} - \sqrt{x}\frac{\partial}{\partial x}(x+1)}{(x+1)^2}$

$= \frac{(x+1)\left(\frac{1}{2}x^{-1/2}\right) - \sqrt{x}}{(x+1)^2}$

$\frac{(4+1)\left(\frac{1}{2}(4)^{-1/2}\right) - \sqrt{4}}{(4+1)^2} = -\frac{3}{100}$

Tangent: $y - 0.4 = -\frac{3}{100}(x - 4)$

Normal: $y - 0.4 = \frac{100}{3}(x - 4)$

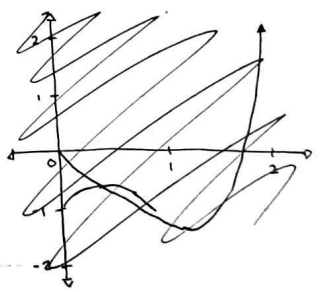
64. a. $s = t^4 - 2t^3 + t^2 - t$

$v(t) = 4t^3 - 6t^2 + 2t - 1$

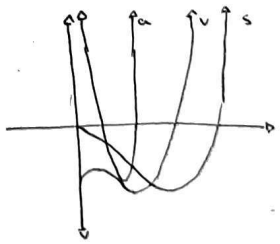
$a(t) = 12t^2 - 12t + 2$

b. $12(1)^2 - 12(1) + 2 = 2\ m/s^2$

c.



c.



66. $S(A) = 0.882A^{0.842}$

$S'(A) = (0.882)(0.842)A^{0.842-1}$

$= 0.742644A^{-0.158}$

$S'(100) = 0.742644(100)^{-0.158} = \boxed{0.358741 \text{ sq/species}}$

This represents the rate of change of tree species at A square meters.

70. a. $h(x) = 3f(x) + 8g(x)$

$h'(x) = 3f'(x) + 8g'(x)$

$h'(4) = 18 + (-24) = \boxed{-6}$

b. $h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + f'(x)g(x)$

$h'(4) = (2)(-3) + (6)(5) = \boxed{24}$

c. $h(x) = \dfrac{f(x)}{g(x)}$

$h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$h'(4) = \dfrac{(5)(6) - (2)(-3)}{5^2} = \boxed{\dfrac{36}{25}}$

d. $h(x) = \dfrac{g(x)}{f(x) + g(x)}$

$h'(x) = \dfrac{(f(x) + g(x))g'(x) - g(x)(f'(x) + g'(x))}{[f(x) + g(x)]^2}$

$h'(4) = \dfrac{(2+5)(-3) - (5)(6-3)}{[2+5]^2} = \boxed{-\dfrac{36}{49}}$