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AP Calc AB: 2.4B HW

32. a. $g(x) = f(x)\sin x$

$g'(x) = f(x)\cdot\frac{d}{dx}\sin x + \sin x\, f'(x)$

$= f(x)\cdot\cos x + \sin x\cdot f'(x)$

$g'(\frac{\pi}{3}) = 4\cos\frac{\pi}{3} - 2\sin\frac{\pi}{3}$

$\boxed{= 2-\sqrt{3}}$

b. $h(x) = \frac{\cos x}{f(x)}$

$h'(x) = \frac{f(x)\cdot\frac{d}{dx}\cos x - \cos x\cdot f'(x)}{[f(x)]^2}$

$= \frac{-f(x)\sin x - \cos x\cdot f'(x)}{[f(x)]^2}$

$h'(\frac{\pi}{3}) = \frac{-4\sin\frac{\pi}{3} + 2\cos\frac{\pi}{3}}{16}$

$\boxed{= \frac{-2\sqrt{3}-1}{16}}$

34. $y = \frac{\cos x}{2+\sin x}$

$y' = \frac{(2+\sin x)\frac{d}{dx}\cos x - \cos x\cdot\frac{d}{dx}[2+\sin x]}{(2+\sin x)^2}$

$= \frac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2}$

$= \frac{-\sin^2 x - 2\sin x - \cos^2 x}{(2+\sin x)^2}$

If tangent line is horizontal, $y'=0$

$0 = -\sin^2 x - 2\sin x - \cos^2 x$

$0 = -1 - 2\sin x$

$\sin x = -\frac{1}{2}$

$\boxed{x = \frac{7\pi}{6} + 2\pi k,\ \frac{11\pi}{6} + 2\pi k}$

40. $\lim_{x\to 0}\frac{\sin x}{\sin\pi x}$

$= \lim_{x\to 0}\frac{\sin x}{\sin\pi x}\cdot\frac{\pi x}{\pi x}$

$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\pi x}{\sin\pi x}\cdot\frac{1}{\pi}$

$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\pi x}{\sin\pi x}\cdot\lim_{x\to 0}\frac{1}{\pi}$

$= \lim_{x\to 0}\frac{1}{\pi}$

$\boxed{= \frac{1}{\pi}}$

Note: $\lim_{x\to 0}\frac{x}{\sin x} = 1$, as it is simply the inverse of $\lim_{x\to 0}\frac{\sin x}{x}$

42. $\lim_{\theta\to 0}\frac{\cos\theta - 1}{\sin\theta}$

$= \lim_{\theta\to 0}\frac{\cos\theta-1}{\sin\theta}\cdot\frac{\theta}{\theta}$

$= \lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}\cdot\frac{\theta}{\sin\theta}$

$= \lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}\cdot\lim_{\theta\to 0}\frac{\theta}{\sin\theta}$

$\boxed{= 0}$

44. $\lim_{x\to 0}\frac{\sin 3x\,\sin 5x}{x^2}$

$= \lim_{x\to 0}\frac{\sin 3x\,\sin 5x}{x^2}\cdot\frac{3}{3}\cdot\frac{5}{5}$

$= \lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{\sin 5x}{5x}\cdot 15$

$= \lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\lim_{x\to 0}\frac{\sin 5x}{5x}\cdot\lim_{x\to 0}15$

$\boxed{=15}$

46. $\lim_{x \to 0} \csc x \cdot \sin(\sin x)$

$= \lim_{x \to 0} \dfrac{\sin(\sin x)}{\sin x}$

$\boxed{= 1}$

48. $\lim_{x \to 0} \dfrac{\sin(x^2)}{x}$

$= \lim_{x \to 0} \dfrac{\sin(x^2)}{x} \cdot \dfrac{x}{x}$

$= \lim_{x \to 0} \dfrac{\sin(x^2)}{x^2} \cdot x$

$= \lim_{x \to 0} \dfrac{\sin(x^2)}{x^2} \cdot \lim_{x \to 0} x = \boxed{0}$

50. $\lim_{x \to 1} \dfrac{\sin(x-1)}{x^2 + x - 2}$

$= \lim_{x \to 1} \dfrac{\sin(x-1)}{(x-1)(x+2)}$

$= \lim_{x \to 1} \dfrac{\sin(x-1)}{x-1} \cdot \lim_{x \to 1} \dfrac{1}{x+2}$

Function of $\frac{\sin x}{x}$ transformed to the right by 1. ∴

$\lim_{x \to 1} \dfrac{\sin(x-1)}{x-1} = 1$

$= 1 \cdot \frac{1}{3}$

$\boxed{= \frac{1}{3}}$

52. Let $f(x) = x \sin x$

1: $f'(x) = x\cos x + \sin x$

2: $f''(x) = x \cdot -\sin x + \cos x + \cos x = -x\sin x + 2\cos x$

3: $f'''(x) = -x\cos x - 3\sin x$

4: $f^{(4)}(x) = -x \cdot (-\sin x) + \cos x(-1) - 3\cos x = x\sin x - 4\cos x$

5: $f^{(5)}(x) = x\cos x + \sin x + 4\sin x = x\cos x + 5\sin x$

6: $f^{(6)}(x) = -x\sin x + \cos x + 5\cos x = -x\sin x + 6\cos x$

A new cycle starts after 4.

35 % 4 = 3 (follows form of 3)

$\boxed{-x\cos x - 35\sin x}$