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AP Calc AB: 2.5B

45. $y = \cos\sqrt{\sin(\tan \pi x)}$

$y' = -\sin\sqrt{\sin(\tan\pi x)}\ \frac{d}{dx}\sqrt{\sin(\tan\pi x)}$

$= -\sin\sqrt{\sin(\tan\pi x)} \cdot \frac{1}{2}(\sin(\tan\pi x))^{-1/2}\ \frac{d}{dx}\sin(\tan\pi x)$

$= -\frac{1}{2}\sin\sqrt{\sin(\tan\pi x)}\ (\sin(\tan\pi x))^{-1/2}\cos(\tan\pi x) \cdot \frac{d}{dx}\tan\pi x$

$= -\frac{1}{2}\sin\sqrt{\sin(\tan\pi x)}\left[\sin(\tan\pi x)\right]^{-1/2}\cos(\tan\pi x)\sec^2\pi x \cdot \pi$

$\boxed{= -\frac{\pi}{2}\sin\sqrt{\sin(\tan\pi x)}\left[\sin(\tan\pi x)\right]^{-1/2}\cos(\tan\pi x)\sec^2\pi x}$

50. $y = \frac{1}{(1+\tan x)^2}$

$= (1+\tan x)^{-2}$

$\boxed{y' = -2(1+\tan x)^{-3}\sec^2 x}$

$y'' = -2(1+\tan x)^{-3}\frac{d}{dx}\sec^2 x + \sec^2 x\frac{d}{dx}\left[-2(1+\tan x)^{-3}\right]$

$= -2(1+\tan x)^{-3}\left[2\sec x \cdot \sec x\tan x\right] + \sec^2 x\left[\cancel{46}\,6(1+\tan x)^{-4}\sec^2 x\right]$

$\boxed{= 2\sec^2 x\left[-2\tan x(1+\tan x)^{-3} + 3\sec^2 x(1+\tan x)^{-4}\right]}$

60. $6x + 2y = 1$

$2y = -6x + 1$

$y = -3x + \frac{1}{2}$

Perpendicular line will have slope of $\frac{1}{3}$

$y = \sqrt{1+2x}$

$y' = \frac{1}{2}(1+2x)^{-1/2} \cdot 2 = (1+2x)^{-1/2}$

$\frac{1}{3} = \frac{1}{\sqrt{1+2x}}$

$3 = \sqrt{1+2x}$

$9 = 1+2x$

$x = 4$ $\qquad \sqrt{1+2(4)} = 3$

$\boxed{y - 3 = \frac{1}{3}(x-4)}$

52. $y = \sqrt{1+x^3}$

$y' = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2$

$= \frac{1}{2\sqrt{1+x^3}} \cdot 3x^2$

Set $x = 2$

$\frac{1}{2\sqrt{1+2^3}} \cdot 3(2)^2 = 2$

$\boxed{y - 3 = 2(x-2)}$

64. a. $F(x) = f(f(x))$

$F'(x) = f'(f(x)) \cdot f'(x)$

$F'(2) = f'(1) \cdot 5 = \boxed{20}$

b. $G(x) = g(g(x))$

$G'(x) = g'(g(x)) \cdot g'(x)$

$G'(3) = g'(2) \cdot 9 = \boxed{63}$

~~$f(x) = x\,g(x^2)$~~

~~$f'(x) = x \cdot \frac{d}{dx}\left[g(x^2)\right] + g(x^2)$~~

~~$= x \cdot g'(x^2) \cdot 2 + g(x^2)$~~

~~$= 2x\,g'(x^2) + g(x^2)$~~

~~$f''(x) = 2x \cdot \frac{d}{dx}\left[g'(x^2)\right] + g'(x^2) \cdot 2 + \frac{d}{dx}\left[g(x^2)\right]$~~

~~$= 2x \cdot g''(x^2) \cdot 2 + g'(x^2) \cdot 2 + g'(x^2) \cdot 2$~~

~~$= 4x\,g''(x^2) + 4g'(x^2)$~~

70. $f(x) = x\,g(x^2)$

$f'(x) = x \frac{d}{dx} g(x^2) + g(x^2)$

$= x \cdot g'(x^2) \cdot 2x + g(x^2)$

$= 2x^2 \cdot g'(x^2) + g(x^2)$

$f''(x) = 2x^2 \frac{d}{dx} g'(x^2) + \frac{d}{dx} g'(x^2) \cdot 4x + \frac{d}{dx} g(x^2)$

$= 2x^2 \cdot g''(x^2) \cdot 2x + 4x g'(x^2) + g'(x^2) \cdot 2x$

$\boxed{= 4x^3 \cdot g''(x^2) + 6x g'(x^2)}$

72. $F(x) = f(x f(x f(x)))$

$F'(x) = f'(x \cdot f(x f(x))) \cdot \frac{d}{dx}\left[x \cdot f(x f(x))\right]$

$= f'(x \cdot f(x f(x)))\left[x \cdot \frac{d}{dx}\left(f(x f(x))\right) + f(x f(x))\right]$

$= f'(x \cdot f(x f(x)))\left[x \cdot f'(x f(x)) \cdot \frac{d}{dx} x f(x) + f(x f(x))\right]$

$= f'(x \cdot f(x f(x)))\left[x \cdot f'(x f(x))\left[x f'(x) + f(x)\right] + f(x f(x))\right]$

$F'(1) = f'(f(f(1)))\left[f'(f(1))\left[f'(1) + f(1)\right] + f(f(1))\right]$

$= f'(f(2))\left[f'(2)\left[4 + 2\right] + f(2)\right]$

$= f'(3)\left[5 \cdot 6 + 3\right]$

$= 6\left[33\right]$

$\boxed{= 198}$

74. $f(x) = x \cdot \sin(\pi x)$

1: $f'(x) = x \cdot \frac{d}{dx} \sin(\pi x) + \sin(\pi x)$

$= x \cdot \cos(\pi x) \cdot \pi + \sin(\pi x)$

$= \pi x \cos(\pi x) + \sin(\pi x)$

2: $f''(x) = \pi x \cdot \frac{d}{dx} \cos(\pi x) + \cos(\pi x) \cdot \pi + \frac{d}{dx} \sin(\pi x)$

$= \pi x \cdot -\sin(\pi x) \cdot \pi + \pi \cos(\pi x) + \pi \cos(\pi x)$

$= -\pi^2 x \cdot \sin(\pi x) + 2\pi \cos(\pi x)$

3: $f'''(x) = -\pi^2 x \frac{d}{dx} \sin(\pi x) + \sin(\pi x) \cdot -\pi^2 + 2\pi \frac{d}{dx}\cos(\pi x)$

$= -\pi^2 x \cos(\pi x) \cdot \pi - \pi^2 \sin(\pi x) - 2\pi \sin(\pi x)\,\pi$

$= -\pi^3 x \cos(\pi x) - 3\pi^2 \sin(\pi x)$

4: $f^{(4)} = -\pi^3 x \cdot \frac{d}{dx} \cos(\pi x) + \cos(\pi x) \cdot (-\pi^3) - 3\pi^2 \frac{d}{dx} \sin(\pi x)$

$= -\pi^3 x \cdot (-\sin \pi x) \cdot \pi - \pi^3 \cos \pi x - 3\pi^2 \cos(\pi x) \cdot \pi$

$= \pi^4 x \sin(\pi x) - 4\pi^3 \cos \pi x$

5: $f^{(5)} = \pi^4 x \frac{d}{dx} \sin(\pi x) + \sin(\pi x) \cdot \pi^4 - 4\pi^3 \frac{d}{dx} \cos \pi x$

$= \pi^4 x \cos(\pi x) \cdot \pi + \sin(\pi x) \cdot \pi^4 - 4\pi^3 (-\sin \pi x)\,\pi$

$= \pi^5 x \cos(\pi x) + 5\pi^4 \sin(\pi x)$

Pattern repeats with every 4. Therefore 35 mod 4 = 3. It follows pattern 3.

$\boxed{-\pi^{35} x \cos(\pi x) - 35\pi^{34} \sin(\pi x)}$