James Dozy

AP Calc AB: HW 2.6A

8. $x^3 - xy^2 + y^3 = 1$

$\frac{d}{dx}\left[x^3 - xy^2 + y^3\right] = \frac{d}{dx} 1$

$0 = 3x^2 - \left(x \cdot \frac{d}{dx} y^2 + y^2\right) + \frac{d}{dx} y^3$

$= 3x^2 - \left(x \cdot 2y \frac{dy}{dx} + y^2\right) + 3y^2 \cdot \frac{dy}{dx}$

$= 3x^2 - 2xy \frac{dy}{dx} - y^2 + 3y^2 \cdot \frac{dy}{dx}$

$-3x^2 + y^2 = -2xy \cdot \frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2 + y^2}{-2xy + 3y^2}}$$

12. $\cos(xy) = 1 + \sin y$

$\frac{d}{dx} \cos(xy) = \frac{d}{dx}\left[1 + \sin y\right]$

$-\sin(xy) \cdot \frac{d}{dx} xy = \cos y \cdot \frac{dy}{dx}$

$-\sin(xy)\left[x \cdot \frac{dy}{dx} + y\right] = \cos y \cdot \frac{dy}{dx}$

$-x\sin(xy) \frac{dy}{dx} + \left(-y \sin(xy)\right) = \cos y \cdot \frac{dy}{dx}$

$-y\sin(xy) = \cos y \cdot \frac{dy}{dx} + x\sin(xy) \cdot \frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = -\frac{y\sin(xy)}{\cos y + \sin xy}}$$

16. $xy = \sqrt{x^2 + y^2}$

$x \frac{dy}{dx} + y = \frac{x + y \cdot \frac{dy}{dx}}{\sqrt{x^2 + y^2}}$

$x\sqrt{x^2+y^2} \frac{dy}{dx} + y\sqrt{x^2+y^2} = x + y \cdot \frac{dy}{dx}$

$y\sqrt{x^2+y^2} - x = y \cdot \frac{dy}{dx} - x\sqrt{x^2+y^2} \cdot \frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{y\sqrt{x^2+y^2} - x}{y - x\sqrt{x^2+y^2}}}$$

20. $\tan(x-y) = \frac{y}{1+x^2}$

$\sec^2(x-y) \frac{d}{dx}\left[x-y\right] = \frac{(1+x^2)\frac{dy}{dx} - y\frac{d}{dx}\left[1+x^2\right]}{(1+x^2)^2}$

$\sec^2(x-y)\left[1 - \frac{dy}{dx}\right] = \frac{(1+x^2)\frac{dy}{dx} - 2yx}{(1+x^2)^2}$

$(1+x^2)^2 \sec^2(x-y)\left[1 - \frac{dy}{dx}\right] = (1+x^2)\frac{dy}{dx} - 2yx$

$(1+x^2)^2 \sec^2(x-y) - (1+x^2)^2 \sec^2(x-y)\frac{dy}{dx} = (1+x^2)\frac{dy}{dx} - 2yx$

$(1+x^2)^2 \sec^2(x-y) + 2yx = (1+x^2)\frac{dy}{dx} + (1+x^2)^2 \sec^2(x-y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{(1+x^2)^2 \sec^2(x-y) + 2xy}{\cancel{(1+x^2)} +}$

$$\boxed{\frac{dy}{dx} = \frac{(1+x^2)^2 \sec^2(x-y) + 2xy}{(1+x^2) + (1+x^2)^2 \sec^2(x-y)}}$$

26 $\sin(x+y) = 2x - 2y$

$\cos(x+y)\left[1 + \frac{dy}{dx}\right] = 2 - 2\frac{dy}{dx}$

$\cos(x+y) + \cos(x+y) \cdot \frac{dy}{dx} = 2 - 2\frac{dy}{dx}$

$\cos(x+y)\frac{dy}{dx} + 2\frac{dy}{dx} = 2 - \cos(x+y)$

$\frac{dy}{dx} = \frac{2 - \cos(x+y)}{\cos(x+y) + 2}$

$\frac{2 - \cos(\pi+\pi)}{\cos(\pi+\pi) + 2} = \frac{1}{3}$

$$\boxed{y - \pi = \frac{1}{3}(x - \pi)}$$

28. $x^2 + 2xy + 4y^2 = 12$

$2x + 2(x\frac{dy}{dx} + y) + 8y \cdot \frac{dy}{dx} = 0$

$2x + 2x\frac{dy}{dx} + 2y + 8y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 8y\frac{dy}{dx} = -2x - 2y$

$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 8y}$

$\frac{-2(2) - 2(1)}{2(2) + 8(1)} = -\frac{1}{2}$

$\boxed{y - 1 = -\frac{1}{2}(x - 2)}$

30. $x^{2/3} + y^{2/3} = 4$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}} \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$

$= \frac{\sqrt[3]{1}}{\sqrt[3]{-3\sqrt{3}}} = \frac{1}{\sqrt{3}}$

$\boxed{y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3})}$

32. $y^2(y^2 - 4) = x^2(x^2 - 5)$

$y^2\frac{d}{dx}[y^2 - 4] + [y^2 - 4] \cdot 2y\frac{dy}{dx} = x^2\frac{d}{dx}[x^2 - 5] + [x^2 - 5] \cdot 2x$

$y^2 \cdot 2y\frac{dy}{dx} + (y^2 - 4)\, 2y\frac{dy}{dx} = x^2 \cdot 2x + (x^2 - 5) \cdot 2x$

$\frac{dy}{dx} = \frac{2x^3 + 2x^3 - 10x}{2y^3 + 2y^3 - 8y} = \frac{4x^3 - 10x}{4y^3 - 8y}$

$= 0$

$y + 2 = 0$; $\boxed{y = -2}$

34. a. $y^2 = x^3 + 3x^2$

$2y \cdot \frac{dy}{dx} = 3x^2 + 6x$

$\frac{dy}{dx} = \frac{3x^2 + 6x}{2y}$

$\frac{3(1) + 6(1)}{2(-2)} = -\frac{9}{4}$

$\boxed{y + 2 = -\frac{9}{4}(x - 1)}$

b. $3x^2 + 6x = 0$

$x(3x + 6) = 0$

$x = 0, -2$ · cannot be at zero — tangent DNE.

Horizontal tangents exist when $x = -2$

$\boxed{(-2, 2) \text{ and } (-2, -2)}$

c.