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AP calc AB: HW 26 B

36. $\frac{\partial}{\partial x}\left[x^2+xy+y^2\right] = \frac{\partial}{\partial x} 3$

$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$

$y'' = \frac{\partial}{\partial x}\left[\frac{-2x-y}{x+2y}\right]$

$= \frac{(x+2y)(-2x-y)' - (-2x-y)(x+2y)'}{(x+2y)^2}$

$= \frac{(x+2y)\left(-2-\frac{dy}{dx}\right) - (-2x-y)\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$

$= \frac{(x+2y)\left(-2+\frac{2x+y}{2y+x}\right) + (2x+y)\left(1+2\cdot\frac{-2x-y}{x+2y}\right)}{(x+2y)^2}$

$= \frac{-3y + (2x+y)\left(1+2\cdot\frac{-2x-y}{x+2y}\right)}{(x+2y)^2}$

$= \frac{-3y + (2x+y)\left(1-\frac{4x+2y}{x+2y}\right)}{(x+2y)^2}$

$= \frac{-3y + (2x+y)\cdot\frac{x}{x+2y} - \frac{(2x+y)(4x+2y)}{x+2y}}{(x+2y)^2}$

$= \frac{-3y(x+2y) + 2x(x+2y) + y(x+2y) - (2x+y)(4x+2y)}{(x+2y)^3}$

$= \frac{-3xy - 6y^2 + 2x^2 + 4xy + xy + 2y^2 - 8x^2 - 8xy - 2y^2}{(x+2y)^3}$

$= \boxed{\frac{-6x^2 - 6xy - 6y^2}{(x+2y)^3}}$

38. $x^3 - y^3 = 7$

$\frac{d}{dx}\left[x^3 - y^3\right] = \frac{d}{dx} 7$

$3x^2 - 3y^2 \frac{dy}{dx} = 0$

$3y^2 \frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{3y^2} = \frac{x^2}{y^2}$

$y'' = \frac{d}{dx}\left[\frac{x^2}{y^2}\right]$

$= \frac{y^2 \frac{d}{dx} x^2 - x^2 \frac{d}{dx} y^2}{y^4}$

$= \frac{2y^2 x - 2x^2 y \cdot \frac{dy}{dx}}{y^4}$

$= \frac{2y^2 x - 2x^2 y \left(\frac{x^2}{y^2}\right)}{y^4}$

$= \frac{2y^2 x - \frac{2x^4}{y}}{y^4}$

$\boxed{= \frac{2y^3 x - 2x^4}{y^5}}$

40. $x^2 + xy + y^3 = 1$

$1 + y + y^3 = 1$

$y + y^3 = 0$

$y = 0$

$\frac{d}{dx}\left[x^2 + xy + y^3\right] = \frac{d}{dx} 1$

$2x + x \frac{dy}{dx} + y + 3y^2 \frac{dy}{dx} = 0$

$2(1) + \frac{dy}{dx} + 0 + 0 = 0$

$\frac{dy}{dx} = -2$

$\frac{d}{dx}\left[2x + x \frac{dy}{dx} + y + 3y^2 \frac{dy}{dx}\right] = \frac{d}{dx} 0$

$2 + x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} + 3y^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} 6y \frac{dy}{dx} = 0$

$2 + \frac{d^2y}{dx^2} - 2 - 2 + 0 + 0 = 0$

$\frac{d^2y}{dx^2} = 2$

$\frac{d}{dx}\left[2 + x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 3y^2 \frac{d^2y}{dx^2} + 6y \left(\frac{dy}{dx}\right)^2\right] = \frac{d}{dx} 0$

$x \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 2 \frac{d^2y}{dx^2} + 3y^2 \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} \cdot 6y \frac{dy}{dx} + 6y \cdot 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \cdot 6 = 0$

$\frac{d^3y}{dx^3} + 2 + 4 + 0 + 0 + 0 + 2^2 \cdot 6 \cdot -2 = 0$

$\frac{d^3y}{dx^3} - 42 = 0$

$\boxed{\frac{d^3y}{dx^3} = 42}$

58. a. $x^2 - xy + y^2 = 3$

$\frac{d}{dx}\left[x^2 - xy + y^2\right] = \frac{d}{dx}3$

From #36

$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$

$\frac{-2(-1) - 1}{-1 + 2(1)} = 1$

$y - 1 = -(x+1)$

$y = -x - 1 + 1 = -x$

$\begin{cases} x^2 - xy + y^2 = 3 \\ y = -x \end{cases}$

$x^2 - x(-x) + (-x)^2 = 3$

$x^2 + x^2 + x^2 = 3$

$3x^2 = 3$

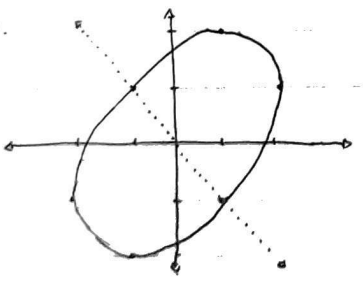
$x = \pm 1$ — known intercept at $(-1, 1)$

$\therefore$ other intercept when $x = 1$

$y = -1$

$(1, -1)$ is the other point the normal line intersects

b.



60. $\frac{d}{dx}\left[x^2 + 4y^2\right] = \frac{d}{dx}36$

$2x + 8y\frac{dy}{dx} = 0$

$8y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{x}{4y}$

Let (a, b) be a point on the equation $x^2 + 4y^2 = 36$

$y - 3 = -\frac{a}{4b}(x - 12)$

$b - 3 = -\frac{a}{4b}(a - 12)$

$-4b(b-3) = a^2 - 12a$

$-4b^2 + 12b = a^2 - 12a$

$12a + 12b = a^2 + 4b^2$

$12(a+b) = 36$

$a + b = 3$

$b = 3 - a$

$a^2 + 4(3-a)^2 = 36$

$a^2 + 4(9 - 6a + a^2) = 36$

$a^2 + 36 - 24a + 4a^2 = 36$

$5a^2 - 24a + 36 = 36$

$5a^2 - 24a = 0$

$a(5a - 24) = 0$

$a = 0, \frac{24}{5}$

$0 + 4y^2 = 36$

$y^2 = 9$

$y = 3$

$\left(\frac{24}{5}\right)^2 + 4y^2 = 36$

$\left(\frac{576}{25}\right) + 4y^2 = 36$

$4y^2 = \frac{324}{25}$

$y^2 = \frac{81}{25}$

$y = \frac{9}{5}$

$m = -\frac{0}{4 \cdot 3} = 0$

$m = -\frac{24/5}{4 \cdot 9/5} = \frac{2}{3}$

$y - 3 = 0(x - 0)$

$y = 3$

$y - 3 = \frac{2}{3}(x - 12)$