James Ding

AP Calc AB: HW 2.7

4. a. $\frac{d}{dt}\left[\frac{9t}{t^2+9}\right]$

$= 9 \cdot \frac{d}{dt}\left[\frac{t}{t^2+9}\right]$

$= 9 \cdot \frac{(t^2+9) - t(2t)}{(t^2+9)^2}$

$= 9 \cdot \frac{t^2+9-2t^2}{(t^2+9)^2}$

$\boxed{= 9 \cdot \frac{-t^2+9}{(t^2+9)^2}}$

b. $9 \cdot \frac{-1+9}{(1+9)^2}$

$\boxed{= 0.72 \text{ fps}}$

c. $0 = 9 \cdot \frac{-t^2+9}{(t^2+9)^2}$

$= -t^2+9$

$t = \pm 3 \; ; \; t \geq 0$

$\boxed{t = 3 \text{ seconds}}$

d. $9 \cdot \frac{-t^2+9}{(t^2+9)^2} > 0$

$-(t+3)(t-3) > 0$

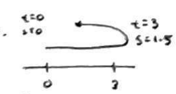
$t \in (-3, 3)$, $t \geq 0$

$\boxed{t \in [0, 3) \text{ seconds}}$

e. $|f(3) - f(0)| + |f(6) - f(3)|$

$\boxed{= \frac{9}{5} \text{ feet}}$

f.



g. $\frac{d}{dt}\left[9 \cdot \frac{-t^2+9}{(t^2+9)^2}\right]$

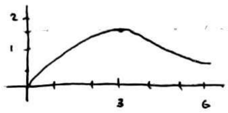
$= 9 \cdot \frac{d}{dt}\left[\frac{-t^2+9}{(t^2+9)^2}\right]$

$= 9 \cdot \frac{(t^2+9)^2(-2t) - (-t^2+9)(2(t^2+9))(2t)}{(t^2+9)^4}$

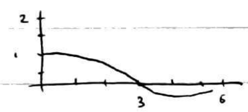
$= 9 \cdot \frac{2t(t^2-27)(t^2+9)}{(t^2+9)^4}$

$\boxed{= \frac{18t(t^2-27)}{(t^2+9)^3}}$ At 1s: $-\frac{117}{250} = \boxed{-0.468 \text{ fps}^2}$

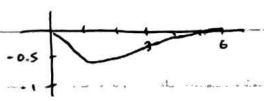
h. Position



Velocity



Accel.



i. Slowing down: $(0,3) \cup (3\sqrt{3}, \infty)$

Speeding up: $(3, 3\sqrt{3})$

6. a. Slowing down: $(0,1) \cup (2,3)$

Speeding up: $(1,2) \cup (3,4)$

b. Slowing down: $(0,1) \cup (2,3)$

Speeding up: $(1,2) \cup (3,4)$

8. a. $-16t^2 + 80t$

$x = -\frac{b}{2a} = \frac{80}{32} = 2.5$

$-16(2.5)^2 + 80(2.5) = 100$

$\boxed{100 \text{ feet}}$

b. $\frac{d}{dt}\left[-16t^2 + 80t\right]$

$= -32t + 80$

$96 = -16t^2 + 80t$

$0 = -16t^2 + 80t - 96$

$= -16(t-2)(t-3)$

$t = 2, 3$

$-32(2) + 80 = 16$ fps

$-32(3) + 80 = -16$ fps

$\boxed{\pm 16 \text{ fps}}$

13. a. i. $3^2\pi - 2^2\pi$

$= 9\pi - 4\pi$

$\boxed{= 5\pi}$

ii. $\dfrac{2.5^2\pi - 2^2\pi}{2.5-2}$

$\boxed{= \dfrac{9\pi}{2}}$

iii. $\dfrac{2.1^2\pi - 2^2\pi}{2.1-2}$

$\boxed{= \dfrac{41\pi}{10}}$

b. $\dfrac{d}{dr}\pi r^2$

$= 2\pi r$

$2\pi(2) = \boxed{4\pi}$

c. $\Delta A = \pi(r+\Delta r)^2 - \pi r^2$

$= \pi\left([r+\Delta r]^2 - r^2\right)$

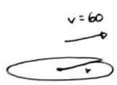
$= \pi\left(r^2 + 2r\Delta r + \Delta r^2 - r^2\right)$

$= \pi\left(2r\Delta r + \Delta r^2\right)$

$\lim_{\Delta r \to 0} \Delta A = 2\pi r \cdot \Delta r$

circumference

14.



$\dfrac{dr}{dt} = 60$ cm/s , solve for $\dfrac{dA}{dt}$

$A = \pi r^2$

$\dfrac{d}{dt}A = \dfrac{d}{dt}\pi r^2$

$\dfrac{dA}{dt} = 2\pi r \cdot \dfrac{dr}{dt}$

a. 120π

b. 360π

c. 600π

The rate that the area increases by increases as time increases.

20. a. $F = \dfrac{GmM}{r^2}$

$\dfrac{dF}{dr} = GmM \dfrac{d}{dr} r^{-2}$

$= GmM \cdot [-2r^{-3}]$

$= -2GmM r^{-3}$

$\dfrac{dF}{dr}$ describes the rate of change of the magnitude of gravitational force on an object.

The negative sign indicates a weaker force as radius increases.

b. $-2GmM(20{,}000)^{-3} = 2$

$GmM = -800.....$

$-2(800...)(10{,}000)^{-3} = 16$

$\boxed{\text{Increases by } 8x}$

24. a. $\dfrac{d}{dt}\left[\dfrac{a^2kt}{akt+1}\right]$

$= \dfrac{(akt+1)\frac{d}{dt}[a^2kt] - a^2kt\frac{d}{dt}[akt+1]}{(akt+1)^2}$

$= \dfrac{(akt+1)a^2k - (a^2kt)ak}{(akt+1)^2}$

$= \dfrac{a^2k[akt+1-akt]}{(akt+1)^2}$

$\boxed{= \dfrac{a^2k}{(ak+1)^2}}$

b. $x = \dfrac{a^2kt}{ak+1}$

$= k \cdot \dfrac{a^2}{(ak+1)^2}$

$= k \cdot \left[\dfrac{a^2kt}{akt+1} - \dfrac{a^2kt}{akt+1} + \dfrac{a}{akt+1}\right]^2 \cdot k$

$= k\left[a - \dfrac{a^2kt}{akt+1}\right]^2$

$= k(a-x)^2$ ✓

30. a. $C'(100)$ represents ~~[illegible]~~ the rate of change of the price at $q=100$

b. $C'(100)$ is 0.13 while the cost for the 101st item is 97.1303. This means it costs 0.13 more to produce ~~[illegible]~~ another item

34. $\frac{d}{dr} 2\sqrt{Dr}$

$= 2 \cdot \frac{1}{2} (Dr)^{-\frac{1}{2}} \cdot D$

$\boxed{= \frac{D}{\sqrt{Dr}}}$ Represents the rate of change of wave speed at r.

36. a. 0.

b. $0.05\left(1 - \frac{P(t)}{10,000}\right) P(t) - 0.04\,P(t) = 0$

$0.05\,P(t) - \frac{5}{1000000} P(t)^2 - 0.04\,P(t) = 0$

$0.01\,P(t) - \frac{5}{1000000} P(t)^2 = 0$

$P(t)\left[0.01 - \frac{5}{1000000} P(t)\right] = 0$

$\boxed{P(t) = 0,\ 2000}$

c. $0.05\,P(t) - \frac{5}{1000000} P(t)^2 - 0.05\,P(t) = 0$

$-\frac{5}{1000000} P(t)^2 = 0$

$\boxed{\text{Only at } 0}$ — it's unsustainable