AP Calc AB: 2.8A

2. a. $A = \pi r^2$

$\frac{d}{dt} A = \frac{d}{dt}\left[\pi r^2\right]$

$\boxed{\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}}$

b. $\frac{dA}{dt} = 2\pi(30)(1)$

$\boxed{= 60\pi \text{ m}^2/\text{s}}$

3. $A = s^2$

$\frac{d}{dt} A = \frac{d}{dt} s^2$

$\frac{dA}{dt} = 2s \cdot \frac{ds}{dt}$

$s^2 = 16 \; ; \; s = 4$

$\frac{dA}{dt} = 2(4)(6)$

$\boxed{= 48 \text{ cm}^2/\text{s}}$

4. $A = \ell w$

$\frac{d}{dt} A = \frac{d}{dt}\left[\ell w\right]$

$\frac{dA}{dt} = \ell \cdot \frac{dw}{dt} + w \cdot \frac{d\ell}{dt}$

$= (20)(3) + (10)(8)$

$\boxed{= 140 \text{ cm}^2/\text{s}}$

5. $V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dV}{dt} \cdot \frac{1}{\pi r^2}$

$= (3) \cdot \frac{1}{25\pi}$

$\boxed{= \frac{3}{25\pi} \text{ m/min}}$

6. $V = \frac{4}{3}\pi r^3$

$\frac{d}{dt} V = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$

$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \cdot \frac{dr}{dt}$

$= 4\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi\left(\frac{80}{2}\right)^2 \cdot (4)$

[redacted]

$\boxed{= 25600\pi \text{ mm}^3/\text{s}}$

12. $xy = 8$

$\frac{d}{dt}\left[xy\right] = 0$

$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$

$(4)(-3) + (2)\frac{dx}{dt} = 0$

$\boxed{\frac{dx}{dt} = 6 \text{ cm/s}}$

16. $a^2 + b^2 = c^2$

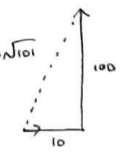
$\frac{d}{dt}\left[a^2 + b^2\right] = \frac{d}{dt} c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$c\frac{dc}{dt} = a\frac{da}{dt} + b\frac{db}{dt}$

$\frac{dc}{dt} = \frac{a\frac{da}{dt} + b\frac{db}{dt}}{c}$

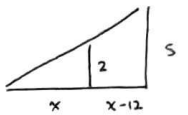
After 4 hours:



$\frac{dc}{dt} = \frac{10(-35) + 100(25)}{10\sqrt{101}}$

$\boxed{= \frac{215}{\sqrt{101}} \text{ km/h}}$

18.



$\frac{2}{x} = \frac{s}{12}$

$x \cdot s = 24$

$s = \frac{24}{x}$

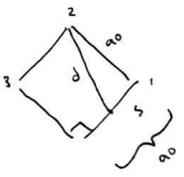
$\frac{d}{dt} s = 24 \cdot \frac{d}{dt}\frac{1}{x}$

$\frac{ds}{dt} = -\frac{24}{x^2} \cdot \frac{dx}{dt}$

$\frac{ds}{dt} = -\frac{24}{64} \cdot (1.6)$

$\boxed{= -0.6 \text{ m/s}}$

20. a.



$s^2 + 90^2 = d^2$

$\frac{d}{dt}\left[s^2 + 90^2\right] = \frac{d}{dt} d^2$

$2s \cdot \frac{ds}{dt} = 2d\frac{dd}{dt}$

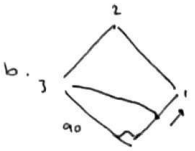
$s \cdot \frac{ds}{dt} = d \cdot \frac{dd}{dt}$

$45(-24) = 45\sqrt{5} \cdot \frac{dd}{dt}$

$-\frac{24}{\sqrt{5}} = \frac{dd}{dt}$

$\boxed{= -\frac{24\sqrt{5}}{5} \text{ ft/s}}$

b.



Same as a, but velo is positive

$\boxed{= \frac{24\sqrt{5}}{5} \text{ ft/s}}$