Jung
Di

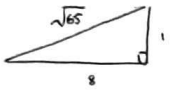
AP Calc AB: 2.8B

22. $a^2+b^2=c^2$

$\frac{d}{dt}\left[a^2+b^2\right]=\frac{d}{dt}c^2$

$2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$

$a\frac{da}{dt}+b\frac{db}{dt}=c\frac{dc}{dt}$



$8\cdot\frac{da}{dt}+0=\sqrt{65}(1)$

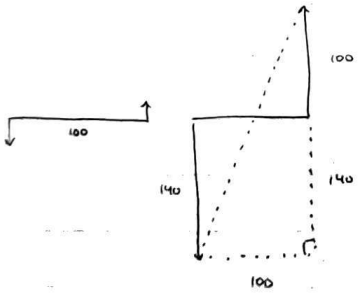
$\boxed{\frac{da}{dt}=-\frac{\sqrt{65}}{8}\ m/s}$

23. $a^2+b^2=c^2$

$\frac{d}{dt}\left[a^2+b^2\right]=\frac{d}{dt}c^2$

$2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$

$a\frac{da}{dt}+b\frac{db}{dt}=c\frac{dc}{dt}$

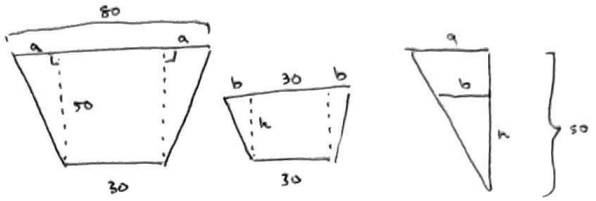


$(240)(25+35)+0=260\frac{dc}{dt}$

$\frac{240\cdot60}{260}=\frac{dc}{dt}$

$\frac{dc}{dt}=\boxed{\frac{720}{13}\ km/h}$

27.



$80-2a=30$

$a=25$

$\frac{25}{50}=\frac{b}{h}$

$b=\frac{1}{2}h$

$V=\frac{(0.3)+(0.3+2b)}{2}\cdot h\cdot 10$

$=\frac{0.6+h}{2}\cdot h\cdot 10$

$=\left(0.3+\frac{h}{2}\right)\cdot h\cdot 10$

$=5h^2+3h$

$\frac{d}{dt}V=\frac{d}{dt}\left[5h^2+3h\right]$

$\frac{dv}{dt}=10h\cdot\frac{dh}{dt}+3\frac{dh}{dt}$

$0.2=10(0.3)\frac{dh}{dt}+3\frac{dh}{dt}$

$0.2=6\frac{dh}{dt}$

$\boxed{\frac{dh}{dt}=\frac{1}{30}\ m/min}$

29. $V = \pi r^2 \frac{h}{3}$

$r = \frac{h}{2}$

$= \pi\left(\frac{h}{2}\right)^2\left(\frac{h}{3}\right)$

$= \frac{\pi}{12}h^3$

$\frac{d}{dt}V = \frac{d}{dt}\left[\frac{\pi}{12}h^3\right]$

$\frac{dV}{dt} = \frac{\pi}{12}\,3h^2 \cdot \frac{dh}{dt}$

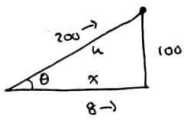
$= \frac{\pi}{4}h^2 \cdot \frac{dh}{dt}$

$30 = \frac{\pi}{4}(10)^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{30}{\frac{\pi}{4}(100)}$

$\boxed{= \frac{6}{5\pi}\ \text{ft/min}}$

30.



$x^2 + 100^2 = h^2$

$x = \sqrt{200^2 - 100^2} = 100\sqrt{3}$

$\sin\theta = \frac{100}{200} = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\tan\theta = \frac{100}{x}$

$\frac{d}{dt}\tan\theta = \frac{d}{dt}\left[\frac{100}{x}\right]$

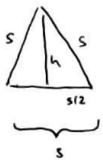
$\sec^2\theta \cdot \frac{d\theta}{dt} = -100 \cdot \frac{1}{x^2} \cdot \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{-100}{x^2} \cdot \cos^2\theta \cdot \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{-100}{(100\sqrt{3})^2} \cdot \cos^2\frac{\pi}{6} \cdot 8$

$\boxed{= -\frac{1}{50}\ \text{ft/s}}$

31.



$h^2 + \left(\frac{s}{2}\right)^2 = s^2$

$h = \sqrt{s^2 - \left(\frac{s^2}{s}\right)}$

$h = \frac{1}{2}\sqrt{4s^2 - s^2}$

$= \frac{\sqrt{3}}{2}s$

$A = \frac{1}{2}bh$

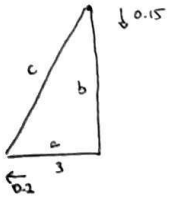
$= \frac{\sqrt{3}}{4}s^2$

$\frac{d}{dt}A = \frac{d}{dt}\left[\frac{\sqrt{3}}{4}s^2\right]$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2}s \cdot \frac{ds}{dt} = \frac{\sqrt{3}}{2}(30)(10)$

$\boxed{= 150\sqrt{3}\ \text{cm}^2/\text{min}}$

33. $a^2 + b^2 = c^2$



$\frac{d}{dt}\left[a^2 + b^2\right] = \frac{d}{dt}c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$a\frac{da}{dt} + b\frac{db}{dt} = c\frac{dc}{dt}$

$3(0.2) + b(-0.15) = 0$

$b = 4$

$a = 3$ $\quad \therefore \boxed{c = 5}$

39. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

$R^{-1} = R_1^{-1} + R_2^{-1}$

$\frac{\partial}{\partial t} R^{-1} = \frac{\partial}{\partial t}\left[R_1^{-1} + R_2^{-1}\right]$

$-R^{-2} \cdot \frac{dR}{dt} = -R_1^{-2} \cdot \frac{dR_1}{dt} + -R_2^{-2} \cdot \frac{dR_2}{dt}$

$\frac{dR}{dt} = \frac{R_1^{-2} \cdot \frac{dR_1}{dt} + R_2^{-2} \frac{dR_2}{dt}}{R^{-2}}$

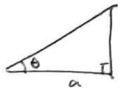
$= \frac{107}{810}\ \Omega/s$

41. $c = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$\frac{dc}{dt} = \frac{1}{2}\left(A^2+B^2-2AB\cos\theta\right)^{-1/2}\left(2AB\sin\theta \cdot \frac{d\theta}{dt}\right)$

$= \frac{1}{2}\left(12^2+15^2-2(12)(15)\cos\frac{\pi}{3}\right)^{-1/2}\left(2(12)(15)\sin\frac{\pi}{3} \cdot \frac{\pi}{90}\right)$

$= \frac{\pi\sqrt{7}}{21}$ m/min

45. 

$\frac{\sin\frac{\pi}{3}}{5} = \frac{\sin\frac{\pi}{6}}{x}$

$x = \frac{5}{\sqrt{3}}$

$\tan\theta = 5a^{-1}$

$\frac{d}{dt}\tan\theta = \frac{d}{dt} 5a^{-1}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = -5a^{-2} \cdot \frac{da}{dt}$

$\frac{da}{dt} = -\frac{\sec^2\theta \cdot \frac{d\theta}{dt}}{5a^{-2}}$

$= \frac{\sec^2\left(\frac{\pi}{3}\right)\left(-\frac{\pi}{6}\right)}{-5\left(\frac{5\sqrt{3}}{3}\right)^{-2}} = \frac{10\pi}{9}$ Km/m

46. 

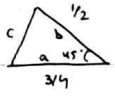
$\sin\theta = \frac{h}{10}$

$h = \sin\theta \cdot 10$

$\frac{dh}{dt} = 10\cos\theta \cdot \frac{d\theta}{dt}$

$= 10\left(\frac{8}{10}\right) \cdot \pi$

$= 8\pi$ m/min

48. 

$20 = \frac{\pi}{90}$

$c = \sqrt{a^2+b^2-2ab\cos\theta}$

$\frac{dc}{dt} = \frac{1}{2}\left(a^2+b^2-2ab\cos\theta\right)^{-1/2}\left(2a\frac{da}{dt} + 2b\frac{db}{dt}\right)$

$\frac{dc}{dt} = \frac{1}{2}\left(a^2+b^2-2ab\cos\theta\right)^{-1/2}\left(2a\frac{da}{dt} + 2b\frac{db}{dt} - 2\cdot\frac{\sqrt{2}}{2}\left(a\frac{db}{dt} + b\frac{da}{dt}\right)\right)$

$= \sqrt{13-6\sqrt{2}}$ mi/h