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AP Calc 3.2

6. $f(x) = x^3 - 2x^2 - 4x + 2$

Because $f(x)$ is a polynomial function
$\therefore f(x)$ is continuous when $x \in [-2, 2]$
and $f(x)$ is differentiable when $x \in (-2, 2)$

$f(-2) = -6$ $\quad f(2) = 6$ $\quad \therefore f(-2) = f(2)$

$f'(x) = 3x^2 - 4x - 4$

$\therefore$ By rolles thrm.

$0 = 3x^2 - 4x - 4$

$\boxed{x = \frac{2 \pm 4}{3}}$

12. $f(x) = x^3 - 3x + 2$

Because $f(x)$ is a polynomial function
$\therefore f(x)$ is continuous when $x \in [-2, 2]$
and differentiable when $x \in (-2, 2)$

$$m = \frac{f(2) - f(-2)}{4} = 1$$

$f'(x) = 3x^2 - 3$

$\therefore$ By MVT

$1 = 3x^2 - 3$

$4 = 3x^2$

$\boxed{x = \pm \frac{2}{\sqrt{3}}}$

16. $f(x) = x^3 - 2x$

Because $f(x)$ is a polynomial function, it is continuous when $x \in [-2, 2]$ and differentiable when $x \in (-2, 2)$

$$m = \frac{f(2) - f(-2)}{4} = 2$$

$f'(x) = 3x^2 - 2$

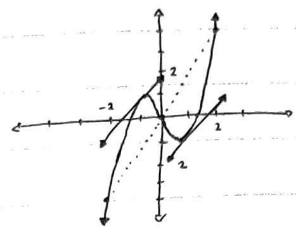
$\therefore$ By MVT

$2 = 3x^2 - 2$

$4 = 3x^2$

$x = \pm \frac{2}{\sqrt{3}} \approx \pm 1.1547$

16. (cont'd)



Secant lines are parallel with tangent

19. $f(0) = 1$
$f(-\frac{\pi}{3}) =$ Negative

Because $f(x)$ is continuous at $\mathbb{R}$, we can apply IVT. Therefore there is atleast one root as $f(x)$ must cross $y = 0$.

Suppose for the sake of contradiction that f has atleast two roots, call them a and b, where $a \neq b$. Then,

$$f(a) = 0 \text{ and } f(b) = 0$$

Because f is comprised of functions that are continuous and differentiable at $\mathbb{R}$, we can apply MVT.

There is a number $c \in (a, b)$ s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= 0$$

$f'(x) = -\sin x + 2$

$\sin x = 2$ Impossible as $\sin x$ is within $[-1, 1]$

This contradicts $f'(c) = 0$

It follows that our assumption that f has atleast two roots is false. Therefore f has atmost one root

By IVT and MVT, f has only one root ∎

21. Suppose for the sake of contradiction that f has atleast two roots, a and b, where $a \neq b$, Then,

$$f(a) = 0 \quad \text{and} \quad f(b) = 0$$

Because f is a polynomial function, it is continuous and differentiable at $\mathbb{R}$. $\therefore$ We can apply MVT

There is a number $c \in (a,b)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$= 0$$

$$f'(x) = 3x^2 - 15$$

$$0 = 3x^2 - 15$$

$$15 = 3x^2$$

$$x^2 = 5$$

$x = \pm\sqrt{5}$ Impossible as not ingiven range $[-2, 2]$

This contradicts $f'(c) = 0$

It follows that our assumption that f has atleast two roots is false. Therefore f has atmost one root. ∎

25. Given that $f'(x) \geq 2$, this means that f is continuous and differentiable at $x \in (1,4)$ $\therefore$ We can apply MVT.

$$f'(x) = \frac{f(4) - f(1)}{3}$$

$$2 \leq \frac{f(4) - 10}{3}$$

$$6 \leq f(4) - 10$$

$$f(4) \geq 16$$

16 is the smallest $f(4)$ can possibly be

26. Given that $3 \leq f'(x) \leq 5$, this means that f is continuous and differentiable at $x \in (2, 8)$. $\therefore$ We can apply MVT

$$f'(c) = \frac{f(8) - f(2)}{8-2}$$

$$6f'(c) = f(8) - f(2)$$

$$3 \leq f'(x) \leq 5$$

$$18 \leq 6f'(x) \leq 30$$

$$18 \leq f(8) - f(2) \leq 30 \quad ∎$$

27. Because $f'(x) \le 2$, this means that f is continuous and differentiable at $x \in (0,2)$ ∴ we can apply MVT

$$f'(x) = \frac{f(2) - f(0)}{2}$$

$$2 \ge \frac{f(2)+1}{2}$$

$$4 \ge f(2) + 1$$

$$3 \ge f(2)$$ Impossible as $f(2) = 4$

Therefore there is no function that exists

34. Because ~~always~~ the acceleration of a car always exists, we can apply MVT.

Note that time elapsed $= \frac{1}{6}$ hour

$$120 = \frac{50-30}{\frac{1}{6}} \checkmark$$

∴ There exists some time where acceleration = 120

36. Assume for the sake of contradiction that a and b were two fixed points. Because $f'(x)$ exists, f is continuous and differentiable. ∴ we apply MVT

$$f'(c) = \frac{b-a}{b-a} = 1$$ Impossible, as given $f'(x) \neq 1$.

∴ By contradiction, there is at most one fixed point.