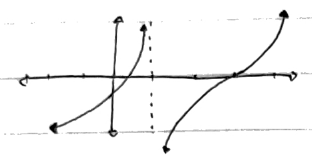
James Ding

AP Calc AB · HW 3.3B

24.

		1		3	
f'	+		+		+
f''	+		−		+
Shape	◡		◠		◡



30. a. B

b. E

c. A

34. a. $f(x) = -2x^3 + 3x^2 + 36x$

$f'(x) = -6x^2 + 6x + 36$

$= -6(x^2 - x - 6)$

$= -6(x-3)(x+2)$

$0 = -6(x-3)(x+2)$

$x = -2, 3$

		−2		3	
f'	−		+		−

Increase: $(-2, 3)$

Decrease: $(-\infty, -2) \cup (3, \infty)$

b.

		−2		3	
f'	−		+		−
		Local Min		Local Max	

$f(-2) = -44$ $f(3) = 81$

Local Min: $(-2, -44)$ Local Max: $(3, 81)$

34. c. $f''(x) = -12x + 6$

$0 = -12x + 6$

$x = \frac{1}{2}$

		$\frac{1}{2}$	
f''	+	Inflection pt.	−

$f(\frac{1}{2}) = \frac{37}{2} = 18.5$

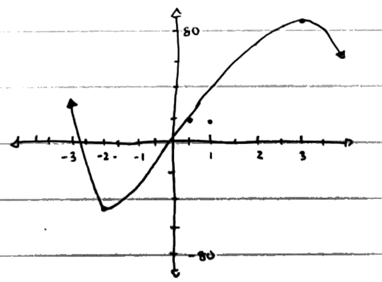
Concave up: $(-\infty, \frac{1}{2})$

Concave down: $(\frac{1}{2}, \infty)$

Inflection point: $(\frac{1}{2}, \frac{37}{2})$

d.

		−2		$\frac{1}{2}$		3	
f'	−		+		+		−
f''	+		+		−		−
Shape	◟		◞		◜		◝



38. a. $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2$

$= -15x^2(x^2 - 1)$

$0 = -15x^2(x^2 - 1)$

$x = 0, \pm 1$

		−1		0		1	
f'	−		+		+		−

Increase: $(-1, 0) \cup (0, 1)$

Decrease: $(-\infty, -1) \cup (1, \infty)$

b.

	−1		0		1	
−		+		+		−
	Local Min				Local Max	

$f(-1) = -2$ $f(1) = 2$

Local Min: $(-1, -2)$ Local Max: $(1, 2)$

c. $f''(x) = -60x^3 + 30x$

$= -30x(2x^2 - 1)$ $f(-\frac{\sqrt{2}}{2}) = -\frac{7\sqrt{2}}{8}$

$0 = -30x(2x^2 - 1)$ $f(0) = 0$

$x = 0, \pm \frac{\sqrt{2}}{2}$ $f(\frac{\sqrt{2}}{2}) = \frac{7\sqrt{2}}{8}$

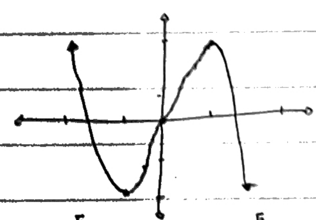
		$-\frac{\sqrt{2}}{2}$		0		$\frac{\sqrt{2}}{2}$	
f''	+		−		+		−

Concave up: $(-\infty, -\frac{\sqrt{2}}{2}) \cup (0, \frac{\sqrt{2}}{2})$

Concave down: $(-\frac{\sqrt{2}}{2}, 0) \cup (\frac{\sqrt{2}}{2}, \infty)$

Inflection points: $(-\frac{\sqrt{2}}{2}, -\frac{7\sqrt{2}}{8})$, $(0, 0)$, $(\frac{\sqrt{2}}{2}, \frac{7\sqrt{2}}{8})$

d.



		−1		$-\frac{\sqrt{2}}{2}$		0		$\frac{\sqrt{2}}{2}$		1	
f'	−		+		+		+		+		−
f''	+		+		−		+		−		−
Shape	◟		◞		◜		◞		◜		◝

64. a. Given f and g are concave up on I, therefore $f''>0$ and $g''>0$. Let $h(x)=f(x)+g(x)$, therefore $h''(x)=f''(x)+g''(x)$ $\therefore$ $f''(x)+g''(x)>0$ given each individual function's second derivative is >0.

b. Given f is concave up on I, $\therefore$ $f''>0$.

$g(x)=[f(x)]^2$

$g'(x)=2f'(x)\cdot f(x)$

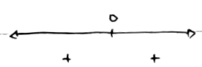
$g''(x)=2\left[f''(x)f(x)+f'(x)\cdot f'(x)\right]$

Since $g''(x)$ is comprised of numbers >0, $\therefore$ it must also be >0, or concave up on I

72. $f(x)=x^4$

$f'(x)=4x^3$

$f''(x)=12x^2$; $f''(0)=0$ ✓



No sign change $\therefore$ Inflection pt. DNE