James Ding

AP Calc AB 3.4

8. $\lim_{x\to\infty} \sqrt{\frac{9x^2+8x-4}{3-5x+x^3}}$

$= \lim_{x\to\infty} \sqrt{\frac{9+8/x^2-4/x^3}{3/x^3-5/x^2+1}}$

$= \sqrt{\frac{9}{1}}$

$= 3$

12. $\lim_{x\to-\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5}$

$= \lim_{x\to-\infty} \frac{4+6/x-2/x^3}{2-4/x^2+5/x^3}$

$= \frac{4}{2}$

$= 2$

16. $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{1+1/x^4}}$

$= 1$

18. $\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$

$= \lim_{x\to-\infty} \frac{-\sqrt{4+1/x^6}}{2/x^3-1}$

$= 2$

22. $\lim_{x\to-\infty} \left[\sqrt{4x^2+3x} + 2x\right]$

$= \lim_{x\to-\infty} \left[\frac{(4x^2+3x)-4x^2}{\sqrt{4x^2+3x}-2x}\right]$

$= \lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2+3x}-2x}$

$= \lim_{x\to-\infty} \frac{3}{\sqrt{4+3/x}-2}$

$= -\frac{3}{4}$

24. $\lim_{x\to\infty} \cos x$

DNE, infinite oscillation

30. $\lim_{x\to\infty} \left[x^2 - x^4\right]$

$= \lim_{x\to\infty} x^4\left(1/x^2 - 1\right)$

$= -\infty$

40. Vertical Asymptotes:

$0 = \sqrt{4x^2+3x+2}$

$= 4x^2+3x+2$

$\frac{-3\pm\sqrt{9-32}}{8} =$ DNE

∴ No vertical asymptotes

Horizontal Asymptotes

$\lim_{x\to\infty} \frac{x-9}{\sqrt{4x^2+3x+2}}$

$= \lim_{x\to\infty} \frac{1-9/x}{\sqrt{4+3/x+2/x^2}}$

$= \frac{1}{2}$

$\lim_{x\to-\infty} \frac{x-9}{\sqrt{4x^2+3x+2}}$

$= \lim_{x\to-\infty} \frac{1-9/x}{-\sqrt{4+3/x+2/x^2}}$

$= -\frac{1}{2}$

Horizontal Asymptotes: $\pm\frac{1}{2}$

46. Vertical Asymptotes at $x=1, 3$

$(x-1)(x-3)$ ∴ Horizontal Asymptote must be x^2

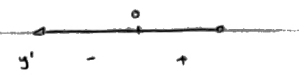
$\frac{x^2}{(x-1)(x-3)}$

48. $\lim_{x\to\infty} \frac{1+2x^2}{1+x^2}$ $\lim_{x\to-\infty} \frac{1+2x^2}{1+x^2}$

$= \lim_{x\to\infty} \frac{1/x^2+2}{1/x^2+1}$ $= \lim_{x\to-\infty} \frac{1/x^2+2}{1/x^2+1}$

$= 2$ $= 2$

$y' = \frac{2x}{(1+x^2)^2}$ $2x = 0 \therefore x = 0$



Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

$y'' = \frac{2-6x^2}{(1+x^2)^3}$ $2-6x^2 = 0$

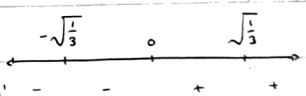
$6x^2 = 2$

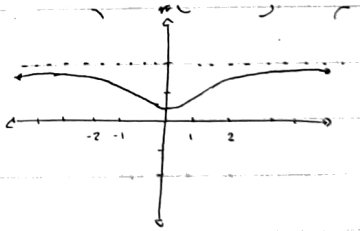
$3x^2 = 1$

$x = \pm\sqrt{\frac{1}{3}}$

concave up: $(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$

Concave down: $(-\infty, -\sqrt{\frac{1}{3}}) \cup (\sqrt{\frac{1}{3}}, \infty)$



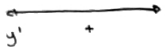


50. $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$ $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{1+1/x^2}}$ $= \lim_{x\to-\infty} \frac{1}{-\sqrt{1+1/x^2}}$

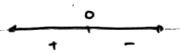
$= 1$ $= -1$

$y' = \frac{1}{(x^2+1)^{3/2}}$



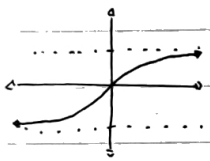
Increases: $(-\infty, \infty)$

$y'' = -\frac{3x}{(x^2+1)^{5/2}}$

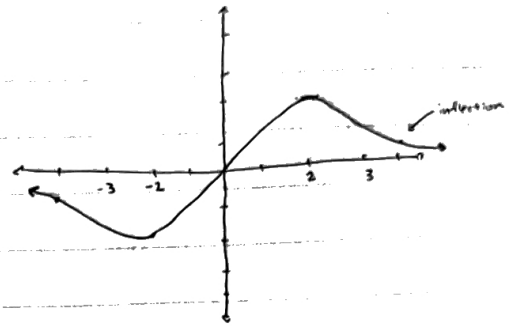


Concave up: $(-\infty, 0)$

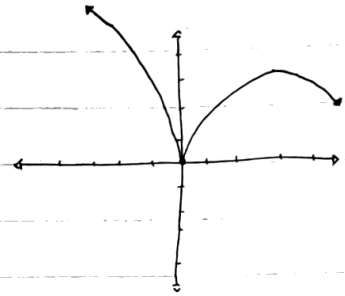
Concave down: $(0, \infty)$



58.



60.



56. $x^2(x^2-1)(x+2)$

$\lim_{x\to\infty} x^2(x^2-1)(x+2) = \infty$

$\lim_{x\to-\infty} x^2(x^2-1)(x+2) = -\infty$

x int: $0, \pm 1, -2$

y int: 0