James Ding

AP Calc AB 3.7.

8.

$1000 = lw \; ; \; w = \frac{1000}{l}$

$P = 2l + 2w$

$= 2l + \frac{2000}{l}$

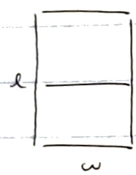
$P' = 2 - \frac{2000}{l^2}$ $\qquad w = \frac{1000}{10\sqrt{10}} = 10\sqrt{10}$

$0 = 2 - \frac{2000}{l^2}$

$l^2 = 1000 \; ; \; l = 10\sqrt{10}$

$\boxed{l = 10\sqrt{10}\text{m} \quad w = 10\sqrt{10}\text{m}}$

13.



$1500\,000 = lw \; ; \; w = \frac{1500\,000}{l}$

$P = 2l + 3w$

$= 2l + \frac{4500\,000}{l}$

$P' = 2 - \frac{4500\,000}{l^2}$ $\qquad w = \frac{1500\,000}{1500} = 1000$

$0 = 2 - \frac{4500\,000}{l^2}$

$l^2 = 2250\,000 \; , \; l = 1500$

$\boxed{l = 1500\text{ ft}, \; w = 1000\text{ ft}}$

21. $D = \sqrt{x^2 + y^2}$ $\qquad y = 2x + 3$

$= \sqrt{x^2 + (2x+3)^2}$

$= \sqrt{x^2 + 4x^2 + 12x + 9}$

$= \sqrt{5x^2 + 12x + 9}$

Find minimum value of D. Because D follows the form of a quadratic, it's vertex is it's lowest point.

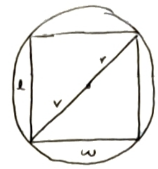
$5x^2 + 12x + 9$

$-\frac{b}{2a} = -\frac{12}{10} = -\frac{6}{5}$

$y = 2\left(-\frac{6}{5}\right) + 3 = \frac{3}{5}$

$\boxed{\left(-\frac{6}{5}, \frac{3}{5}\right)}$

25.



$A = lw$

$(2r)^2 = l^2 + w^2$

$4r^2 = l^2 + w^2$

$w = \sqrt{4r^2 - l^2}$

Find max of A: $\quad A = l\sqrt{4r^2 - l^2}$

$= \sqrt{4l^2r^2 - l^4}$

$\frac{d}{dx}\left[4l^2r^2 - l^4\right]$

$0 = 8r^2 l - 4l^3$

$= -4l(l^2 - 2r^2)$

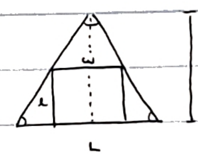
$0 = l^2 - 2r^2$ $\qquad l = 0 \leftarrow$ impossible as $l \in (0, \infty)$

$l^2 = 2r^2$

$l = r\sqrt{2}$ $\qquad w = \sqrt{4r^2 - 2r^2} = \sqrt{2r^2} = r\sqrt{2}$

$\boxed{l = r\sqrt{2}, \; w = r\sqrt{2}}$

27.



$\frac{\frac{\sqrt{3}}{2}L - l}{w} = \frac{\frac{\sqrt{3}}{2}L}{L}$

$\frac{\sqrt{3}}{2}L - l = \frac{\sqrt{3}}{2}w$

$l = \frac{\sqrt{3}}{2}(L - w)$

$A = lw = w\,\frac{\sqrt{3}}{2}(L - w)$

$= \frac{\sqrt{3}}{2}Lw - \frac{\sqrt{3}}{2}w^2$

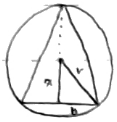
Because A is a quadratic function, it's vertex is the ~~[illegible]~~ max.

$-\frac{b}{2a} = \frac{\sqrt{3}\cdot L}{2\sqrt{3}} = \frac{L}{2} \; ; \; w = \frac{1}{2}L$

$l = \frac{\sqrt{3}}{2}\left(L - \frac{L}{2}\right) = \frac{\sqrt{3}}{4}L$

$\boxed{w = \frac{1}{2}L, \; l = \frac{\sqrt{3}}{4}L}$

29.



$h = x + r$ $\qquad r^2 = x^2 + b^2$

$A = \frac{1}{2}(2b)h$ $\qquad b = \sqrt{r^2 - x^2}$

$= b(x+r)$

$= \sqrt{r^2 - x^2}\,(x+r)$

$$A' = \sqrt{r^2 - x^2} + \frac{1}{2}(x+r)(r^2 - x^2)^{-1/2}(-2x)$$

$$= \sqrt{r^2 - x^2} + \frac{-2x(x+r)}{2\sqrt{r^2 - x^2}}$$

$$= \sqrt{r^2 - x^2} + \frac{-x(x+r)}{\sqrt{r^2 - x^2}}$$

$$= \frac{r^2 - x^2}{\sqrt{r^2 - x^2}} + \frac{-x(x+r)}{\sqrt{r^2 - x^2}}$$

$$0 = r^2 - x^2 - x^2 - xr$$

$$= -2x^2 - xr + r^2 \quad = 2x^2 + xr - r^2$$

$$x = \frac{-r \pm \sqrt{r^2 + 8r^2}}{4}$$

$$= \frac{-r \pm \sqrt{9r^2}}{4}$$

$$= \frac{-r \pm 3r}{4} = \frac{r}{2}$$

$$h = \frac{r}{2} + r = \frac{3r}{2}$$

$$b = \sqrt{r^2 - \left(\frac{r}{2}\right)^2}$$

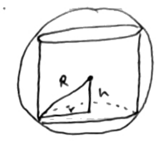
$$= \sqrt{r^2 - \frac{r^2}{4}}$$

$$= \frac{r\sqrt{3}}{2}$$

$$2b = r\sqrt{3}$$

$$\boxed{h = \frac{3r}{2}, \text{ base} = r\sqrt{3}}$$

31.



$h^2 + r^2 = R^2$, $h = \sqrt{R^2 - r^2}$

$V = \pi r^2 (2h)$

$= \pi r^2 \cdot 2\sqrt{R^2 - r^2}$

$$V' = 4\pi r\sqrt{R^2 - r^2} - \frac{2\pi r^3}{\sqrt{R^2 - r^2}}$$

$$0 = \frac{4\pi r(R^2 - r^2)}{\sqrt{R^2 - r^2}} - \frac{2\pi r^3}{\sqrt{R^2 - r^2}}$$

$$0 = 4\pi r(R^2 - r^2) - 2\pi r^3$$

$$= 4\pi r R^2 - 4\pi r^3 - 2\pi r^3$$

$$= 4\pi R^2 - 6\pi r^3$$

$$6\pi r^3 = 4\pi r R^2$$

$$6\pi r^2 = 4\pi R^2$$

$$r^2 = \frac{2}{3}R^2$$

$$r = \sqrt{\frac{2}{3}}\,R$$

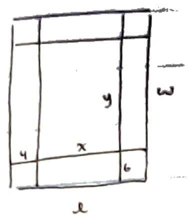
$$\pi\left(\sqrt{\frac{2}{3}}R\right)^2 \cdot 2\sqrt{R^2 - \left(\sqrt{\frac{2}{3}}R\right)^2}$$

$$= \pi \cdot \frac{2}{3}R^2 \cdot 2\sqrt{\frac{1}{3}R^2}$$

$$\boxed{= \frac{4\pi R^3}{3\sqrt{3}}}$$

41.

35.



$384 = xy$

$y = \frac{384}{x}$ $\qquad$ $l = x + 8$

$\qquad\qquad$ $w = y + 12$

$A = lw$

$= (x+8)(y+12)$

$= xy + 12x + 8y + 96$

$= 384 + 12x + \frac{384 \cdot 8}{x} + 96$

$= 12x + \frac{384 \cdot 8}{x} + 480$

$A' = -3072x^{-2} + 12$

$0 = -3072x^{-2} + 12$

$12 = 3072x^{-2}$

$12x^2 = 3072$

$x^2 = 256$

$x = 16$; $y = \frac{384}{16} = 24$

$l = 16 + 8 = 24$

$w = 24 + 12 = 32$

$\boxed{l = 24,\ w = 32}$

37. x $\qquad$ $10 - x$

$A = \left(\frac{x}{4}\right)^2 + \left(\frac{10-x}{3}\right)^2 \cdot \frac{\sqrt{3}}{4}$

$= \frac{x^2}{16} + \frac{\sqrt{3}\,(10-x)^2}{9 \cdot 4}$

$= \frac{9x^2 + 4\sqrt{3}\,(10-x)^2}{144}$

$= \frac{9x^2 + 4\sqrt{3}\,(x^2 - 20x + 100)}{144}$

$= \frac{9x^2 + 4\sqrt{3}\,x^2 - 80\sqrt{3}\,x + 400\sqrt{3}}{144}$

$= \frac{(9 + 4\sqrt{3})x^2 - 80\sqrt{3}\,x + 400\sqrt{3}}{144}$

a. A follows form of quadratic. ∴ use endpoints to find max within $x \in [0, 10]$

$A(10) > A(0)$ ∴ $\boxed{\text{cut square to 10}}$

b. $x = -\frac{b}{2a} = \frac{80\sqrt{3}}{2(9+4\sqrt{3})} = \boxed{\frac{40\sqrt{3}}{9+4\sqrt{3}}}$