AP Calc 3.8

13. a. $f(x) = 3x^4 - 8x^3 + 2$, continuous at $\mathbb{R}$

$3(2)^4 - 8(2)^3 + 2 < 0$
$3(3)^4 - 8(3)^3 + 2 > 0$

$\therefore$ By IVT, there exists a root within the interval $[2,3]$

b. $f(x) = 3x^4 - 8x^3 + 2$
$f'(x) = 12x^3 - 24x^2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_1 = 3$, $x_2 = 2.73148$, $x_3 = 2.64044$, $x_4 = 2.63015$,
$x_5 = 2.63002$, $x_6 = 2.63002$

$\boxed{= 2.63002}$

17. $f(x) = 3\cos x - x - 1$
$f'(x) = -3\sin x - 1$

Roots near $-4, -2, 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_1 = -4$, $x_2 = -3.68228$, $x_3 = -3.63896$, $x_4 = -3.63796$
$x_5 = -3.63796$

$x_1 = -2$, $x_2 = -1.85622$, $x_3 = -1.86236$, $x_4 = -1.86236$

$x_1 = 1$, $x_2 = 0.892438$, $x_3 = 0.889473$, $x_4 = 0.88947$,
$x_5 = 0.88947$

$\boxed{-3.63796, \ -1.86236, \ 0.88947}$

19. $f(x) = \sqrt[3]{x} - \frac{1}{x} - 1$
$f'(x) = \frac{1}{3}x^{-2/3} + x^{-2}$

Roots near $-1, 3$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_1 = -1$, $x_2 = -0.25$, $x_3 = -0.390739$, $x_4 = -0.50619$
$x_5 = -0.546524$, $x_6 = -0.549684$, $x_7 = -0.5497$

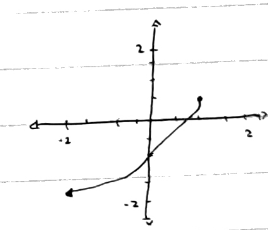
$x_1 = 3$, $x_2 = 2.59863$, $x_3 = 2.62942$, $x_4 = 2.62966$
$x_5 = 2.62966$

$\boxed{-0.5497, \ 2.62966}$

25. $f(x) = \frac{x}{x^2+1} - \sqrt{1-x}$

$f'(x) = \frac{x^2+1-2x^2}{(x^2+1)^2} - \frac{1}{2}(1-x)^{-1/2} \cdot -1$

$= \frac{-x^2+1}{(x^2+1)^2} + \frac{1}{2}(1-x)^{-1/2}$



Root near 0.5

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_1 = 0.5$, $x_2 = 0.758702$, $x_3 = 0.766859$, $x_4 = 0.766826$
$x_5 = 0.766826$

$\boxed{0.766826}$

29. $f(x) = x^3 - 3x + 6$
$f'(x) = 3x^2 - 3$

At $x=1$, derivative is 0 and won't cross the x-axis (tangent line)