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AP Calculus AB

HW 4.1c - Riemann Sum

1. Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the table below.

Time(h)	0	1	2	3	4	5	6	7	8
Leakage (gal/hr)	50	70	97	136	190	265	369	516	720

a) Give an upper and lower estimate of the total quantity of oil that has escaped after 5 hours.

$L_5 = 50 + 70 + 97 + 136 + 190 =$ 543 gallons $\quad R_5 = 70 + 97 + 136 + 190 + 265 =$ 758 gallons

b) Give an upper and lower estimate of the total quantity of oil that has escaped after 8 hours.

$L_8 = L_5 + 265 + 369 + 516 =$ 1693 gallons $\quad R_8 = R_5 + 369 + 516 + 720 =$ 2363 gallons

c) The tanker continues to leak 720 gallons/hour after the first 8 hours. If the tanker originally contained 25,000 gallons of oil, approximately how many more hours will elapse in the worst case before all of the oil has leaked? in the best case?

Worst Case

$\frac{25000 - 2362}{720} =$ 31.44 hours

Best Case

$\frac{25000 - 1693}{720} =$ 32.37 hours

2. The table shows speedometer readings for a motorcycle at 12-second intervals.

Time (s)	0	12	24	36	48	60
Vel (ft/s)	30	28	25	22	24	27

a) Estimate the distance traveled by the motorcycle during this time period using the velocities at the **beginning** of the time intervals.

$12(30 + 28 + 25 + 22 + 24) =$ 1548 ft

b) Give another estimate using the velocities at the **end** of the time periods.

$12(28 + 25 + 22 + 24 + 27) =$ 1512 ft

3. Find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

a) $f(x) = x^2 + \sqrt{1+2x}$, $x \in [4, 7]$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right)\left(\left[4+\frac{3i}{n}\right]^2 + \sqrt{1+2\left[4+\frac{3i}{n}\right]}\right)$$

b) $g(x) = 1 - x^9$, $x \in [2, 5]$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right)\left(1-\left[2+\frac{3i}{n}\right]^2\right)$$

c) $h(x) = \sin 3\pi x$, $x \in [0, 1]$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{1}{n}\right)\left(\sin\left(\frac{3\pi i}{n}\right)\right)$$

d) The area under the curve $y = x^5$ from 0 to 2.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left(\frac{2i}{n}\right)^5$$

e) The area under the curve $y = x^4 + 5x^2 + x$ from 2 to 7.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5}{n}\right)\left[\left(2+\frac{5i}{n}\right)^4 + 5\left(2+\frac{5i}{n}\right)^2 + 2 + \frac{5i}{n}\right]$$

4. Describe the following plane regions:

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$

$b-a = 3$ $\quad a = 1$

$f(x) = \sqrt{x}$ $\quad b = 4$

Area under $\sqrt{x}$ between interval $x \in [1, 4]$

b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{2n} \sin\frac{i\pi}{2n}$

$b-a = \pi/2$ $\quad a = 0$

$f(x) = \sin x$ $\quad b = \pi/2$

Area under $\sin x$ between $x \in [0, \frac{\pi}{2}]$

c) $\left(\sqrt{2+\frac{1}{n}} + \sqrt{2+\frac{2}{n}} + \sqrt{2+\frac{3}{n}} + \ldots + \sqrt{2+\frac{n}{n}}\right)\left(\frac{1}{n}\right)$

$b-a = 1$ $\quad b = 3$

$a = 2$ $\quad f(x) = \sqrt{x}$

Area under $\sqrt{x}$ between $x \in [2, 3]$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{8}{n} \ln\left(2+\frac{8i}{n}\right)$

$b-a = 8$ $\quad b = 10$

$a = 2$ $\quad f(x) = \ln x$

Area under $\ln x$ between $x \in [2, 10]$