Math Homework 4.2 A

6. a. $R_6 = -1.5 + 1.5 + 0.5 - 1 + 0.5$
$\boxed{= 0}$

b. $L_6 = -1.5 + 1.5 + 0.5 - 1$
$\boxed{= -0.5}$

c. $M_6 = -1 - 1 + 1 + 1 - 0.5$
$\boxed{= -0.5}$

7. $L = 4(-12 - 6 - 2 + 1 + 3) = -16$
$R = 4(-6 - 2 + 1 + 3 + 8) = 16$

Underestimate: $\boxed{-16}$
~~Overestimate~~: $\boxed{16}$

8. a. $2(-0.6 + 0.9 + 1.8) = 4.2$
Greater than exact

b. $2(-3.4 - 0.6 + 0.9) = -6.2$
Less than exact

c. $2(-2.1 + 0.3 + 1.4) = -1.6$
Less than exact

10. $\int_0^1 \sqrt{x^3+1}\, dx \;;\; n = 5 \;\therefore \Delta x = \frac{1}{5}$

$= \frac{1}{5}\left(f\left(\frac{1}{10}\right) + f\left(\frac{3}{10}\right) + f\left(\frac{5}{10}\right) + f\left(\frac{7}{10}\right) + f\left(\frac{9}{10}\right)\right)$

$\boxed{= 1.1096}$

18. $\int_2^5 \left(x\sqrt{1+x^3}\right) dx$

20. $\int_1^3 \left(\frac{x}{x^2+4}\right) dx$

22. $\int_1^4 (x^2 - 4x + 2)\, dx$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right)\left[\left(1+\frac{3i}{n}\right)^2 - 4\left(1+\frac{3i}{n}\right) + 2\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right)\left[\frac{9i^2}{n^2} + \frac{6i}{n} + 1 - 4 - \frac{12i}{n} + 2\right]$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{i=1}^{n} \left[\frac{9i^2}{n^2} - \frac{6i}{n} - 1\right]$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)\left[\sum_{i=1}^{n} \frac{9i^2}{n^2} - \sum_{i=1}^{n} \frac{6i}{n} - \sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)\left[\frac{9}{n^2} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{6}{n} \cdot \frac{n(n+1)}{2} - n\right]$$

$$= \lim_{n\to\infty} \left[\frac{27n(n+1)(2n+1)}{6n^3} - \frac{18n(n+1)}{2n^2} - 3\right]$$

$$= \lim_{n\to\infty} \left[\frac{27(n+1)(2n+1)}{6n^2} - \frac{18(n+1)}{2n} - 3\right]$$

$= 9 - 9 - 3 = \boxed{-3}$

24. $\int_0^2 (2x - x^3)$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left[2\left(\frac{2i}{n}\right) - \left(\frac{2i}{n}\right)^3\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left[\frac{4i}{n} - \frac{8i^3}{n^3}\right]$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right)\left[\frac{4}{n} \cdot \frac{n(n+1)}{2} - \frac{8}{n^3} \cdot \frac{n^2(n+1)^2}{4}\right]$$

$$= \lim_{n\to\infty} \left[\frac{8(n+1)}{2n} - \frac{16(n+1)^2}{4n^2}\right]$$

$$= \lim_{n\to\infty} \left[\frac{4(n+1)}{n} - \frac{4(n+1)^2}{n^2}\right]$$

$= 4 - 4 = \boxed{0}$