Jamy
Diz

Math Homework 4.2B

28. Assuming $a=0$

$$\int_0^b x^2\,dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b}{n}\right)\left(\frac{bi}{n}\right)^2$$

$$= \lim_{n\to\infty} \frac{b^3}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \left(\frac{b^3}{n^3}\right)\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \frac{b^3}{3}$$

$$\int_a^b x^2dx = \int_a^0 x^2dx + \int_0^b x^2dx$$

$$\int_a^0 x^2dx = -\int_0^a x^2dx$$

$$= -\frac{a^3}{3}$$

$$\int_a^b x^2dx = \frac{b^3}{3} - \frac{a^3}{3}$$

$$= \frac{b^3-a^3}{3}$$ ●

34. a. $\int_0^2 g(x)\,dx$

$$= \frac{1}{2}(2)(4)$$

$$\boxed{= 4}$$

b. $\int_2^6 g(x)\,dx$

$$= -\frac{1}{2}\pi(2)^2$$

$$\boxed{= -2\pi}$$

c. $\int_0^7 g(x)\,dx$

$$= \int_0^2 g(x)dx + \int_2^6 g(x)dx + \int_6^7 g(x)dx$$

$$= 4 - 2\pi + \frac{1}{2}$$

$$\boxed{= \frac{9}{2} - 2\pi}$$

35. $\int_{-1}^{2} (1-x)\,dx$

$$= \int_{-1}^{1} (1-x)\,dx + \int_{1}^{2} (1-x)\,dx$$

$$= 2 - \frac{1}{2}$$

$$\boxed{= \frac{3}{2}}$$

36. $\int_0^6 \left(\frac{1}{3}x-2\right)dx + \int_6^9 \left(\frac{1}{3}x-2\right)dx$

$$= -6 + \frac{3}{2}$$

$$\boxed{= -\frac{9}{2}}$$

37. $\int_{-3}^{0} \left(1+\sqrt{9-x^2}\right)dx$

$$= \int_{-3}^{0} 1 + \int_{-3}^{0} \sqrt{9-x^2}$$

$$= 3 + \frac{1}{4}\pi(3)^2$$

$$\boxed{= \frac{9\pi}{4} + 3}$$

38. $\int_{-5}^{5} \left(x - \sqrt{25-x^2}\right)dx$

$$= \int_{-5}^{5} x - \int_{-5}^{5} \sqrt{25-x^2}\,dx$$

$$= -\frac{1}{2}\pi(5)^2$$

$$\boxed{= -\frac{25\pi}{2}}$$

39. $\int_{-4}^{3} \left|\frac{1}{2}x\right| dx$

$$= \int_{-4}^{0} \left|\frac{1}{2}x\right| dx + \int_{0}^{3} \left|\frac{1}{2}x\right| dx$$

$$= 4 + \frac{9}{4}$$

$$\boxed{= \frac{25}{4}}$$

40. $\int_0^{1/2} |2x-1|\,dx + \int_{1/2}^{1} |2x-1|\,dx$

$$= \frac{1}{4} + \frac{1}{4}$$

$$\boxed{= \frac{1}{2}}$$

42. $\int_{\pi}^{0} \sin^4\theta\,d\theta = -\int_0^{\pi} \sin^4\theta\,d\theta$

$$\boxed{= -\frac{3\pi}{8}}$$

44. $\int_2^5 (1+3x^4)\,dx$

$$= \int_2^5 1 + \int_2^5 3x^4\,dx$$

$$= 3 + 3\int_2^5 x^4\,dx$$

$$= 3 + 3(618.6)$$

$$\boxed{= 1858.8}$$