James Dluy

Math Homework 4.3

8. $g(x) = \int_1^x \cos(t^2)\,dt$

$\boxed{g'(x) = \cos(x^2)}$

12. $R(y) = \int_y^2 t^3 \sin t\,dt$

$= -\int_2^y t^3 \sin t\,dt$

$\boxed{r(y) = -y^3 \sin y}$

16. $y = \int_0^{x^4} \cos^2\theta\,d\theta$

$\boxed{y' = \cos^2(x^4)\cdot 4x^3}$

20. $\int_{-1}^1 x^{100}\,dx$

$F(x) = \frac{x^{101}}{101}$

$F(1) - F(-1) = \frac{1}{101} + \frac{1}{101}$

$\boxed{= \frac{2}{101}}$

24. $\int_1^8 x^{-2/3}\,dx$

$F(x) = 3x^{1/3}$

$F(8) - F(1) = 3(2) - 3$

$\boxed{= 3}$

28. $\int_0^4 (4-t)\sqrt{t}\,dt$

$= \int_0^4 (4\sqrt{t} - t^{3/2})\,dt$

$= \int_0^4 4\sqrt{t}\,dt - \int_0^4 t^{3/2}\,dt$

$= 4\int_0^4 \sqrt{t}\,dt - \int_0^4 t^{3/2}\,dt$

$F(x) = \frac{8x^{3/2}}{3}$ $\quad G(x) = \frac{2x^{5/2}}{5}$

$= \frac{64}{3} - \frac{64}{5}$

$\boxed{= \frac{128}{15}}$

32. $\int_{\pi/4}^{\pi/3} \csc^2\theta\,d\theta$

$F(x) = -\cot x$

$F(\frac{\pi}{3}) - F(\frac{\pi}{4})$

$\boxed{= 1 - \frac{\sqrt{3}}{3}}$

36. $\int_1^{18} \sqrt{\frac{3}{z}}\,dz$

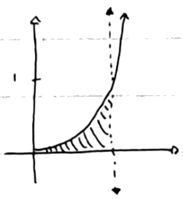
$F(x) = \sqrt{3}\cdot 2x^{1/2}$

$= 2\sqrt{3x}$

$F(18) - F(1)$

$\boxed{= 6\sqrt{6} - 2\sqrt{3}}$

40.



$\int_0^1 x^3\,dx$

$F(x) = \frac{x^4}{4}$

$F(1) - F(0)$

$\boxed{= \frac{1}{4}}$

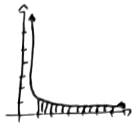
56. $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt$

$g'(x) = -\int_0^{\tan x} \frac{1}{\sqrt{2+t^4}}\,dt + \int_0^{x^2} \frac{1}{\sqrt{2+t^4}}$

$= \frac{1}{\sqrt{2+\tan^4 x}}\cdot\sec^2 x + \frac{1}{\sqrt{2+x^8}}\cdot 2x$

$\boxed{= \frac{\sec^2 x}{\sqrt{2+\tan^4 x}} + \frac{2x}{\sqrt{2+x^8}}}$

44.



Estimate: 0.4

$\int_1^6 x^{-4}\,dx$

$F(x) = -\frac{1}{3x^3}$

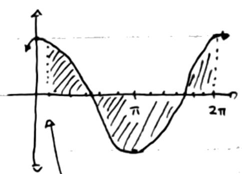
$F(6) - F(1) = \frac{215}{648}$

$\boxed{\approx 0.33179}$

48. $\int_{\pi/6}^{2\pi} \cos x\,dx$

$F(x) = \sin x$

$F(2\pi) - F(\frac{\pi}{6})$



$\int_0^{\pi/6} \cos x\,dx = \frac{1}{2}$

Because there would be an extra $\frac{1}{2}$ for the $\int_0^{2\pi} \cos x\,dx$ compared to $\int_{\pi/6}^{2\pi} \cos x\,dx$, therefore

$\int_{\pi/6}^{2\pi} \cos x\,dx = -\frac{1}{2}$

52. $\sec^2 x$ is discontinuous along the interval $x \in [0, \pi]$.

60. $F(x) = \int_1^x f(t)\,dt$

$F'(x) = f(x)$

$F''(x) = f'(x)$

(when $f'(x)$ is negative)

$\boxed{\therefore x \in (-1, 1)}$

68. $\int_0^1 \sqrt{x}\,dx$

$F(x) = \frac{2x^{3/2}}{3}$

$F(1) - F(0)$

$\boxed{= \frac{2}{3}}$

62. $f(x) = \int_0^{\sin x} \sqrt{1+t^2}\,dt$

$g(y) = \int_3^y f(x)\,dx$

$g'(y) = f(y)$

$g''(y) = f'(y)$

$= \sqrt{1+\sin^2 y} \cdot \cos y$

$g''\left(\frac{\pi}{6}\right)$

$\boxed{= \frac{\sqrt{15}}{4}}$

66. a. Local Min: 4, 8

Local Max: 2, 6

b. 2

c. $g''(x) = f'(x)$

(when $f'(x)$ is negative)

$(1,3) \cup (5,7) \cup (9,10)$

d.