Math Homework 4.5A

10. $\int \sin t \sqrt{1+\cos t}\, dt$

let $u = 1+\cos t$

$\frac{du}{dt} = -\sin t$

$dt = -\frac{du}{\sin t}$

$= \int \sin t \sqrt{u} \cdot -\frac{du}{\sin t}$

$= \int -\sqrt{u}\, du$

$= -\frac{2}{3} u^{3/2} + C$

$= -\frac{2}{3}(1+\cos t)^{3/2} + C$

12. $\int \sec^2 2\theta\, d\theta$

let $u = 2\theta$

$\frac{du}{d\theta} = 2$

$d\theta = \frac{du}{2}$

$= \int \sec^2 u \cdot \frac{du}{2}$

$= \frac{1}{2}\tan u + C$

$= \frac{1}{2}\tan 2\theta + C$

14. $\int y^2 (4-y^3)^{2/3}\, dy$

let $u = 4-y^3$

$\frac{du}{dy} = -3y^2$

$dy = -\frac{1}{3} y^{-2}\, du$

$= \int -\frac{1}{3} u^{2/3}\, du$

$= -\frac{1}{5} u^{5/3} + C$

$= -\frac{1}{5}(4-y^3)^{5/3} + C$

16. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\, du$

$= \int \frac{\sin\sqrt{x}}{\sqrt{x}} \cdot 2\sqrt{x}\, du$

$= \int 2\sin u\, du$

$= -2\cos u + C$

$= -2\cos\sqrt{x} + C$

18. $\int \sin x \sin(\cos x)\, dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

$= \int \sin x \cdot \sin u \cdot -\frac{du}{\sin x}$

$= \int -\sin u\, du$

$= \cos u + C$

$= \cos(\cos x) + C$

20. $\int x\sqrt{x+2}\, dx$

let $u = x+2$ ; $x = u-2$

$\frac{du}{dx} = 1$

$du = dx$

$= \int x\sqrt{u}\, du$

$= \int (u-2)\sqrt{u}\, du$

$= \int (u^{3/2} - 2\sqrt{u})\, du$

$= \frac{2}{5} u^{5/2} - \frac{4}{3} u^{3/2} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

22. $\int \cos\left(\frac{\pi}{x}\right) x^{-2}\, dx$

let $u = \frac{\pi}{x}$

$\frac{du}{dx} = -\frac{\pi}{x^2}$

$dx = -\frac{x^2}{\pi}\, du$

$= \int \cos u \cdot \frac{1}{x^2} \cdot -\frac{x^2}{\pi}\, du$

$= \int -\frac{1}{\pi} \cdot \cos u\, du$

$= -\frac{\sin u}{\pi} + C$

$= -\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right) + C$

24. $\int \frac{1}{\cos^2 t \sqrt{1+\tan t}}\, dt$

let $u = 1+\tan t$

$\frac{du}{dt} = \sec^2 t$

$dt = \frac{du}{\sec^2 t}$

$= \cos^2 t\, du$

$= \int \frac{\cos^2 t}{\cos^2 t \sqrt{u}}\, du$

$= \int u^{-1/2}\, du$

$= 2\sqrt{u} + C$

$= 2\sqrt{1+\tan t} + C$

26. $\int \frac{\sec^2 x}{\tan^2 x}\, dx \Rightarrow \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} \Rightarrow \frac{1}{\sin^2 x}$

$= \int \frac{1}{\sin^2 x}\, dx$

$= -\cot^2 x + C$

28. $\int x^2\sqrt{2+x}\, dx$

let $u = 2+x$ ; $x = u-2$

$\frac{du}{dx} = 1$

$du = dx$

$= \int x^2\sqrt{u}\, du$

$= \int (u-2)^2\sqrt{u}\, du$

$= \int (u^2 - 4u + 4)\sqrt{u}\, du$

$= \int (u^{5/2} - 4u^{3/2} + 4u^{1/2})\, du$

$= \frac{2}{7} u^{7/2} - \frac{8}{5} u^{5/2} + 8u^{3/2} + C$

$= \frac{2}{7}(2+x)^{7/2} - \frac{8}{5}(2+x)^{5/2} + 8(2+x)^{3/2} + C$

30. $\int x^3\sqrt{x^2+1}\,dx$

let $u = x^2+1$ ; $x^2 = u-1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \int x^3\sqrt{u}\cdot\frac{du}{2x}$

$= \int x^2\sqrt{u}\cdot\frac{1}{2}\cdot du$

$= \int \frac{1}{2}(u-1)\sqrt{u}\,du$

$= \int \frac{1}{2}u^{3/2} - \frac{1}{2}u^{1/2}\,du$

$= \frac{1}{5}u^{5/2} - \frac{1}{3}u^{3/2} + C$

$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$