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Math Homework, 4.5 B

36. $\int_0^1 (3t-1)^{50}\,dt$

let $u = 3t-1$

$\frac{du}{dt} = 3$

$dt = \frac{du}{3}$

$= \int_0^1 u^{50}\,\frac{du}{3}$

$= \frac{1}{153}u^{51}\Big|_0^1$

$= \frac{1}{153}(3t-1)^{51}\Big|_0^1$

$= \frac{1}{153}(2^{51}+1)$

38. $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$

let $u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \int_0^{\sqrt{\pi}} x\cos u \cdot \frac{du}{2x}$

$= \int_0^{\sqrt{\pi}} \frac{\cos u}{2}\,du$

$= \frac{\sin u}{2}\Big|_0^{\sqrt{\pi}}$

$= \frac{\sin x^2}{2}\Big|_0^{\sqrt{\pi}}$

$= 0$

40. $\int_{\pi/2}^{\frac{2\pi}{3}} \csc^2\left(\frac{1}{2}t\right)\,dt$

let $u = \frac{1}{2}t$

$\frac{du}{dt} = \frac{1}{2}$

$dt = 2du$

$= \int_{\pi/3}^{2\pi/3} 2\csc^2 u\,du$

$= -2\cot u\Big|_{\pi/3}^{2\pi/3}$

$= -2\cot\left(\frac{1}{2}t\right)\Big|_{\pi/3}^{2\pi/3}$

$= \frac{4\sqrt{3}}{3}$

42. $\int_0^{\pi/2} \cos x\cdot\sin(\sin x)\,dx$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$= \int_0^{\pi/2} \cos x\cdot\sin(\sin x)\cdot\frac{du}{\cos x}$

$= \int_0^{\pi/2} \sin u\,du$

$= -\cos u\Big|_0^{\pi/2}$

$= -\cos(\sin x)\Big|_0^{\pi/2}$

$= 1-\cos(1)$

44. $\int_0^a x\sqrt{a^2-x^2}\,dx$

let $u = a^2-x^2$

$\frac{du}{dx} = -2x$

$dx = -\frac{du}{2x}$

$= \int_0^a x\sqrt{u}\cdot -\frac{du}{2x}$

$= \int_0^a -\sqrt{u}/2\cdot du$

$= -\frac{1}{3}u^{3/2}\Big|_0^a$

$= -\frac{1}{3}(a^2-x^2)^{3/2}\Big|_0^a$

$= \frac{1}{3}a^3$

46. $\int_{-\pi/3}^{\pi/3} x^4\sin x\,dx$

Let $f(x) = x^4\sin x$

$f(x)$ is odd as

$f(-x) = -x^4\sin x$

$\therefore$ Because $a=-b$,

$\int_{-\pi/3}^{\pi/3} x^4\sin x = 0$

48. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$

let $u = 1+2x$; $x = \frac{u-1}{2}$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$= \int_0^9 \frac{x}{\sqrt{u}}\cdot\frac{du}{2}$

$= \int_0^4 \frac{u-1}{4\sqrt{u}}\,du$

$= \int_0^4 \left(\frac{1}{4}u^{1/2} - \frac{1}{4}u^{-1/2}\right)du$

$= \left(\frac{1}{6}u^{3/2} - \frac{1}{2}u^{1/2}\right)\Big|_0^4$

$= \left(\frac{1}{6}(1+2x)^{3/2} - \frac{1}{2}(1+2x)^{1/2}\right)\Big|_0^4$

$= \frac{10}{3}$

50. $\int_0^{T/2} \sin(2\pi t/T-\alpha)\,dt$

let $u = \frac{2\pi t}{T} - \alpha$

$\frac{du}{dt} = \frac{2\pi}{T}$

$dt = \frac{du\cdot T}{2\pi}$

$= \int_0^{T/2} \sin u\cdot\frac{du\,T}{2\pi}$

$= -\cos u\cdot\frac{T}{2\pi}\Big|_0^{T/2}$

$= -\cos\left(\frac{2\pi t}{T}-\alpha\right)\cdot\frac{T}{2\pi}\Big|_0^{T/2}$

$= \cos\alpha\cdot\frac{T}{\pi}$

56. $\int_0^1 x\sqrt{1-x^4}\,dx$

let $u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \int_0^1 \frac{1}{2}\sqrt{1-u^2}\,du$

$= \frac{1}{2}\cdot\frac{\pi}{4}$

$= \frac{\pi}{8}$

60. $\int_0^9 f(x)\,dx = 4$

$\int_0^3 xf(x^2)\,dx$

let $u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \int_0^3 xf(u) \cdot \frac{du}{2x}$

$= \int_0^3 \frac{f(u)}{2}\,du$

$\int_0^9 \frac{f(u)}{2}\,du$

$= \int_0^9 \frac{f(x^2)}{2}\,du$

$\boxed{= 2}$

66. $\int_0^{\pi/2} f(\cos x)\,dx = \int_0^{\pi/2} f(\sin x)\,dx$

$\int_0^{\pi/2} \cos^2 x\,dx$

$= \int_0^{\pi/2} (1 - \sin^2 x)\,dx$

$= \frac{\pi}{2} - \int_0^{\pi/2} \sin^2 x\,dx$

$= \frac{\pi}{2} - \int_0^{\pi/2} \cos^2 x\,dx$

$\int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{2} - \int_0^{\pi/2} \cos^2 x\,dx$

$2\int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{2}$

$\boxed{\int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}}$

$\therefore$

$\boxed{\int_0^{\pi/2} \sin^2 x\,dx = \frac{\pi}{4}}$

72. $\int e^{\cos t} \sin t\,dt$

let $u = \cos t$

$\frac{du}{dt} = -\sin t$

$dt = -\frac{du}{\sin t}$

$= \int e^u \cdot -du$

$= -e^u + C$

$\boxed{= -e^{\cos t} + C}$

76. $\int \frac{\sin(\ln x)}{x}\,dx$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$= \int \frac{\sin u}{x} \cdot x\,du$

$= \int \sin u\,du$

$= -\cos u + C$

$\boxed{= -\cos(\ln x) + C}$

82. $\int_0^1 xe^{-x^2}\,dx$

let $u = -x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$= \int_0^1 -\frac{1}{2} e^u\,du$

$= -\frac{1}{2} e^u \Big|_0^1$

$\boxed{= \frac{1}{2} - \frac{1}{2e}}$

84. $\int_0^2 (x-1)\, e^{(x-1)^2}\,dx$

let $u = (x-1)^2$

$\frac{du}{dx} = 2(x-1)$

$dx = \frac{du}{2(x-1)}$

$= \int_0^2 \frac{1}{2} e^u\,du$

$= \frac{1}{2} e^u \Big|_0^2$

$= \frac{1}{2} e^{(x-1)^2} \Big|_0^2$

$\boxed{= 0}$