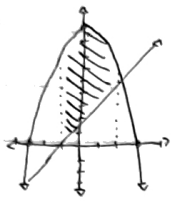
James Ding

Math Homework 5.1

5.



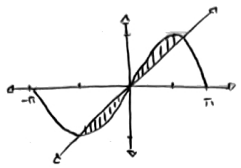
$$\int_{-1}^{2}\left[(9-x^2)-(x+1)\right]dx$$

$$=\int_{-1}^{2}\left(-x^2-x+8\right)dx$$

$$=\left(-\frac{1}{3}x^3-\frac{1}{2}x^2+8x\right)\Big|_{-1}^{2}$$

$$=\frac{39}{2}$$

10.

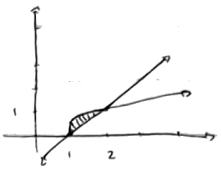


$$\int_0^{\pi/2}\left[\sin x-\frac{2}{\pi}x\right]dx$$

$$=\left(-\cos x-\frac{1}{\pi}x^2\right)\Big|_0^{\pi/2}$$

$$=1-\frac{\pi}{4}$$

18.



$x-y=1$

$y=x-1$

$$\int_1^2\left[\sqrt{x-1}-(x-1)\right]dx$$

$$=\int_1^2\sqrt{x-1}\,dx-\int_1^2(x-1)\,dx$$

let $u=x-1$ $\quad=\left(\frac{1}{2}x^2-x\right)\Big|_1^2$

$\frac{du}{dx}=1$; $dx=du$

$$=\int_1^2\sqrt{u}\,du \Rightarrow \frac{2}{3}(x-1)^{3/2}\Big|_1^2$$

$$=\frac{2}{3}(x-1)^{3/2}\Big|_1^2-\left(\frac{1}{2}x^2-x\right)\Big|_1^2$$

$$=\frac{1}{6}$$

$x^2=4x-x^2$

$2x^2=4x$

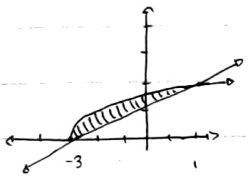
$x^2=2x$

$0=x^2-2x$

$=x(x-2)$

$x=0,2$

9.



$$\int_{-3}^{1}\left[\sqrt{x+3}-\frac{1}{2}(x+3)\right]dx$$

$$=\int_{-3}^{1}\sqrt{x+3}\,dx-\int_{-3}^{1}\left(\frac{1}{2}x+\frac{3}{2}\right)dx$$

let $u=x+3$ $\quad=\left(\frac{1}{4}x^2+\frac{3}{2}x\right)\Big|_{-3}^{1}$

$\frac{du}{dx}=1$, $dx=du$

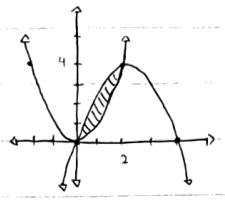
$$=\int_{-3}^{1}\sqrt{u}\,du$$

$$=\frac{2}{3}u^{3/2}\Big|_{-3}^{1}$$

$$=\frac{2}{3}u^{3/2}\Big|_{-3}^{1}-\left(\frac{1}{4}x^2+\frac{3}{2}x\right)\Big|_{-3}^{1}$$

$$=\frac{4}{3}$$

14.



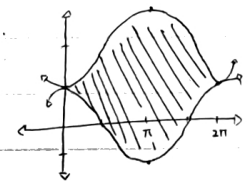
$$\int_0^2\left[4x-x^2-x^2\right]dx$$

$$=\int_0^2\left(4x-2x^2\right)$$

$$=2\left(x^2-\frac{1}{3}x^3\right)\Big|_0^2$$

$$=\frac{8}{3}$$

16.



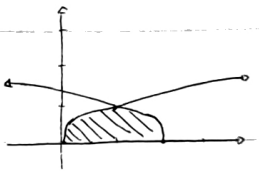
$$\int_0^{2\pi}\left[2-\cos x-\cos x\right]dx$$

$$=\int_0^{2\pi}\left(2-2\cos x\right)dx$$

$$=2\left(x-\sin x\right)\Big|_0^{2\pi}$$

$$=4\pi$$

20.



$x=y^4$

$y=\sqrt[4]{x}$

$$\int_0^2\left[\sqrt{2-x}-\sqrt[4]{x}\right]dx$$

$$=\int_0^2\sqrt{2-x}\,dx-\int_0^2\sqrt[4]{x}\,dx$$

let $u=2-x$ $\quad=\frac{4}{5}x^{5/4}\Big|_0^2$

$\frac{du}{dx}=-1$

$dx=-du$

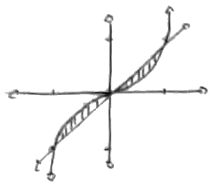
$$\int_0^2\sqrt{u}\cdot-1\,du$$

$$=-\frac{2}{3}u^{3/2}\Big|_0^2$$

$$=-\frac{2}{3}(2-x)^{3/2}\Big|_0^2-\frac{4}{5}x^{5/4}\Big|_0^2$$

$$=\frac{4\sqrt{2}}{3}-\frac{8\sqrt[4]{2}}{5}$$

22.

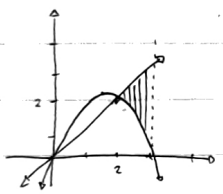


$\int_{-1}^{1} (x - x^3)\,dx$

Because $x - x^3$ is odd, $\int_{-1}^{1} (x - x^3)\,dx$ must be 0 by definition

$= 0$

26.



$\int_2^3 \left(x - (3x - x^2)\right) dx$

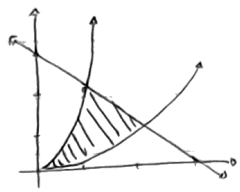
$= \int_2^3 (x - 3x + x^2)\,dx$

$= \int_2^3 (x^2 - 2x)\,dx$

$= \left(\frac{1}{3}x^3 - x^2\right)\Big|_2^3$

$= \frac{4}{3}$

28.



$x + y = 3$

$y = -x + 3$

$\int_0^1 \left[2x^2 - \frac{1}{4}x^2\right] dx + \int_1^2 \left[-x + 3 - \frac{1}{4}x^2\right] dx$

$= \int_0^1 \frac{7}{4}x^2\,dx$ $= \int_1^2 \left(-\frac{1}{4}x^2 - x + 3\right) dx$

$= \frac{7}{12}x^3\Big|_0^1$ $= \left(-\frac{1}{12}x^3 - \frac{1}{2}x^2 + 3x\right)\Big|_1^2$

$= \frac{7}{12}$ $= \frac{11}{12}$

$\frac{7}{12} + \frac{11}{12} = \frac{3}{2}$

30. $\int_0^{10} \left[\left(2200 + 52.3t + 0.74t^2\right) - \left(1460 + 25.8t\right)\right] dt$

$= \int_0^{10} \left(0.74t^2 + 23.5t + 740\right) dt$

$= \left(\frac{0.74}{3}t^3 + \frac{23.5}{2}t^2 + 740t\right)\Big|_0^{10}$

≈ 8822 Represents the change in population which accounts for births/deaths.