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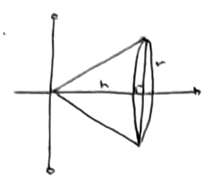
Math Homework 5.2d

47.



$\int_0^h \pi\left(\frac{r}{h}x\right)^2 dx$

$= \pi\int_0^h \frac{r^2}{h^2}\cdot x^2\,dx$

$= \frac{\pi r^2}{h^2}\int_0^h x^2 dx$

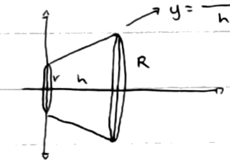
$= \frac{\pi r^2}{h^2}\left[\frac{1}{3}x^3\right]_0^h$

$= \frac{\pi r^2}{h^2}\cdot\frac{1}{3}h^3$

$\boxed{= \frac{1}{3}\pi r^2 h}$

48.

$y = \frac{R-r}{h}x + r$



$= \int_0^h \pi\left(\frac{R-r}{h}x + r\right)^2 dx$

$= \pi\int_0^h \left(\frac{R-r}{h}x + r\right)^2 dx$

let $u = \frac{R-r}{h}x + r$

$\frac{du}{dx} = \frac{R-r}{h}$

$dx = \frac{h}{R-r}du$

$= \pi\int_0^h u^2\cdot\frac{h}{R-r}du$

$= \frac{\pi h}{R-r}\left[\frac{1}{3}u^3\right]_0^h$

$= \frac{\pi h}{R-r}\cdot\left[\frac{1}{3}\left(\frac{R-r}{h}x+r\right)^3\right]_0^h$

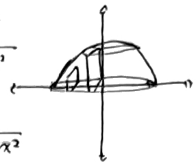
$= \frac{\pi h}{R-r}\cdot\frac{1}{3}(R^3 - r^3)$

$= \frac{\pi h}{3(R-r)}\cdot(R-r)(R^2 + Rr + r^2)$

$\boxed{= \frac{\pi h}{3}(R^2 + Rr + r^2)}$

54. $x^2 + y^2 = r^2$

$y = \sqrt{r^2 - x^2}$



$s = 2y$

$= 2\sqrt{r^2 - x^2}$

$A = s^2$

$= 4(r^2 - x^2)$

$\int_{-r}^{r} 4(r^2 - x^2)\,dx$

$= \int_0^r 8(r^2 - x^2)\,dx$

$= 8\int_0^r (r^2 - x^2)\,dx$

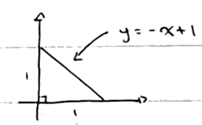
$= 8\left[r^2 x - \frac{1}{3}x^3\right]_0^r$

$= 8\left(r^3 - \frac{1}{3}r^3\right)$

$= 8\left(\frac{2}{3}r^3\right)$

$\boxed{= \frac{16r^3}{3}}$

56.



$A = \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}(-x+1)^2$

$\int_0^1 \frac{\sqrt{3}}{4}(-x+1)^2\,dx$

$= \frac{\sqrt{3}}{4}\int_0^1 (-x+1)^2\,dx$

$= \frac{\sqrt{3}}{4}\int_0^1 (x^2 - 2x + 1)\,dx$

$= \frac{\sqrt{3}}{4}\left[\frac{1}{3}x^3 - x^2 + x\right]_0^1$

$= \frac{\sqrt{3}}{4}\cdot\frac{1}{3}$

$\boxed{= \frac{\sqrt{3}}{12}}$

57.

$A = s^2$

$= (-x+1)^2$

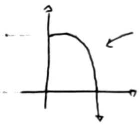
$\int_0^1 (-x+1)^2\,dx$

$= \int_0^1 (x^2 - 2x + 1)\,dx$

$= \left[\frac{1}{3}x^3 - x^2 + x\right]_0^1$

$\boxed{= \frac{1}{3}}$

58.



$s = 2\sqrt{1-y}$

$A = s^2$

$= 4(1-y)$

$\int_0^1 4(1-y)\,dy$

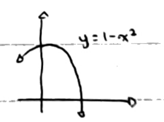
$= 4\int_0^1 (-y+1)\,dy$

$= 4\left[-\frac{1}{2}y^2 + y\right]_0^1$

$= 4\cdot\frac{1}{2}$

$\boxed{= 2}$

59.



$A = \frac{1}{2}bh$

$= \frac{1}{2}(1-x^2)(1-x^2)^2$

$= \frac{1}{2}(1-x^2)^2$

$\int_{-1}^{1}\frac{1}{2}(1-x^2)^2\,dx$

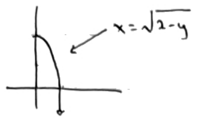
$= \int_{-1}^{1}\frac{1}{2}(1 - 2x^2 + x^4)\,dx$

$= \int_0^1 (1 - 2x^2 + x^4)\,dx$

$= \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_0^1$

$\boxed{= \frac{8}{15}}$

60.



$r = 2\sqrt{2-y}$

$A = \pi r^2$

$A = \frac{\pi}{4}\left(2\sqrt{2-y}\right)^2$

$\int_0^2 \frac{\pi}{4}\left(2\sqrt{2-y}\right)^2 dy$

$= \int_0^2 \pi(2-y)\,dy$

$= \pi\int_0^2 (2-y)\,dy$

$= \pi\left[2y - \frac{1}{2}y^2\right]_0^2$

$\boxed{= 2\pi}$

62. a. $x^2 + y^2 = r^2$

$y = \sqrt{r^2 - x^2}$

$b = 2y$

$= 2\sqrt{r^2-x^2}$

$A = \frac{1}{2}bh$

$= h\sqrt{r^2-x^2}$

$\int_{-r}^{r} \left(h\sqrt{r^2-x^2}\right) dx$

$\boxed{= 2h\int_0^r \left(\sqrt{r^2-x^2}\right) dx}$

63. a $(x-R)^2 + y^2 = r^2$

$(x-R)^2 = r^2 - y^2$

$x - R = \sqrt{r^2-y^2}$

$x = \sqrt{r^2-y^2} + R$

$\int_{-r}^{r} \pi\left[\left(R+\sqrt{r^2-y^2}\right)^2 - \left(R-\sqrt{r^2-y^2}\right)^2\right] dy$

$\boxed{= 2\pi\int_0^r \left[\left(R+\sqrt{r^2-y^2}\right)^2 - \left(R-\sqrt{r^2-y^2}\right)^2\right] dy}$

63. b. $2\pi\int_0^r \left[\left(R+\sqrt{r^2-y^2}\right)^2 - \left(R-\sqrt{r^2-y^2}\right)^2\right] dy$

$= 2\pi\int_0^r \left[\left(R^2 + 2R\sqrt{r^2-y^2} + r^2 - y^2 - \left(R^2 - 2R\sqrt{r^2-y^2} + r^2 - y^2\right)\right)\right] dy$

$= 2\pi\int_0^r 4R\sqrt{r^2-y^2}\,dy$

$= 8\pi R\int_0^r \sqrt{r^2-y^2}\,dy$

$= 8\pi R\left[\frac{1}{2}y\sqrt{r^2-y^2} + \frac{1}{2}\sin^{-1}\left(\frac{y}{r}\right)\right]_0^r$

$= 8\pi R\left(\frac{r^2}{2}\cdot\frac{\pi}{2}\right)$

$= 2\pi^2 r^2 R$