James Ding

AP Calc AB: HW 6.4

2. $\frac{\partial}{\partial x}\left[x\ln x - x\right]$

$= x\frac{\partial}{\partial x}\ln x + \ln x - 1$

$= x\cdot\frac{1}{x} + \ln x - 1$

$= 1 + \ln x - 1$

$\boxed{= \ln x}$

4. $\frac{\partial}{\partial x}\ln(\sin^2 x)$

$= \frac{1}{\sin^2 x}\cdot\frac{\partial}{\partial x}\sin^2 x$

$= \frac{1}{\sin^2 x}\cdot 2\sin x\cdot\frac{\partial}{\partial x}\sin x$

$= \frac{2}{\sin x}\cdot\cos x$

$\boxed{= 2\cot x}$

6. $\frac{\partial}{\partial x}\left[\frac{1}{\ln x}\right] = \frac{\partial}{\partial x}\left[(\ln x)^{-1}\right]$

$= -(\ln x)^{-2}\cdot\frac{\partial}{\partial x}\ln x$

$= -(\ln x)^{-2}\cdot\frac{1}{x}$

$\boxed{= \frac{1}{x(\ln x)^2}}$

8. $\frac{\partial}{\partial x}\log_{10}\sqrt{x}$

$= \frac{1}{\ln(10)\sqrt{x}}\cdot\frac{\partial}{\partial x}\sqrt{x}$

$= \frac{1}{\ln(10)\sqrt{x}}\cdot\frac{1}{2}x^{-1/2}$

$\boxed{= \frac{1}{2x\cdot\ln(10)}}$

10. $\frac{\partial}{\partial t}\sqrt{1+\ln t}$

$= \frac{1}{2}(1+\ln t)^{-1/2}\cdot\frac{\partial}{\partial t}(1+\ln t)$

$= \frac{1}{2}(1+\ln t)^{-1/2}\cdot\frac{1}{t}$

$\boxed{= \frac{1}{2t\sqrt{1+\ln t}}}$

12. $\frac{\partial}{\partial x}\ln(x+\sqrt{x^2-1})$

$= \frac{1}{x+\sqrt{x^2-1}}\cdot\frac{\partial}{\partial x}(x+\sqrt{x^2-1})$

$= \frac{1}{x+\sqrt{x^2-1}}\left(1+\frac{1}{2}(x^2-1)^{-1/2}\cdot 2x\right)$

$= \frac{1}{x+\sqrt{x^2-1}}\cdot\left(1+\frac{x}{\sqrt{x^2-1}}\right)$

$= \frac{1}{x+\sqrt{x^2-1}}\cdot\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}$

$= \frac{\sqrt{x^2-1}+x}{x\sqrt{x^2-1}+x^2-1}$

$\boxed{= \frac{1}{\sqrt{x^2-1}}}$

16. $\frac{\partial}{\partial x}\ln|1+t-t^3|$

$= \frac{1}{1+t-t^3}\cdot\frac{\partial}{\partial t}\left[1+t-t^3\right]$

$= \frac{1}{1+t-t^3}\cdot\left[1-3t^2\right]$

$\boxed{= \frac{1-3t^2}{1+t-t^3}}$

20. $\frac{\partial}{\partial x}\ln(\csc x - \cot x)$

$= \frac{1}{\csc x-\cot x}\cdot\frac{\partial}{\partial x}\left[\csc x-\cot x\right]$

$= \frac{1}{\csc x-\cot x}\cdot\left[-\csc x\cdot\cot x+\csc^2 x\right]$

$\boxed{= \frac{-\csc x\cot x+\csc^2 x}{\csc x-\cot x}}$

24. $\frac{\partial}{\partial x}\log_2(x\log_5 x)$

$\frac{1}{\ln(2)\,x\log_5 x}\cdot\frac{\partial}{\partial x}$

$= \frac{1}{\ln(2)\,x\log_5 x}\cdot\frac{\partial}{\partial x}\left[x\log_5 x\right]$

$= \frac{1}{\ln(2)\,x\log_5 x}\cdot\left(x\frac{\partial}{\partial x}\log_5 x+\log_5 x\right)$

$\frac{1}{\ln(2)\,x\log_5 x}\cdot$

$= \frac{1}{\ln(2)\,x\log_5 x}\cdot\left(x\cdot\frac{1}{\ln(5)x}+\log_5 x\right)$

$= \frac{1}{\ln(2)\,x\log_5 x}\left(\frac{1}{\ln(5)}+\log_5 x\right)$

$\boxed{= \frac{\ln(5)^{-1}+\log_5 x}{\ln(2)\,x\log_5 x}}$

28. $\frac{\partial}{\partial x}\left[\frac{\ln x}{1+\ln x}\right]$

$= \frac{(1+\ln x)\cdot\frac{1}{x}-\ln x\cdot\frac{1}{x}}{(1+\ln x)^2}$

$= \frac{1+\ln x-\ln x}{x(1+\ln x)^2}$

$\boxed{= \frac{1}{x(1+\ln x)^2}}$

$\frac{\partial}{\partial x}\left[x^{-1}\cdot(1+\ln x)^{-2}\right]$

$= \frac{1}{x}\cdot\frac{\partial}{\partial x}(1+\ln x)^{-2}+(1+\ln x)^{-2}\cdot\frac{\partial}{\partial x}x^{-1}$

$= \frac{1}{x}\cdot -2(1+\ln x)^{-3}\cdot\frac{1}{x}+(1+\ln x)^{-2}\cdot -x^{-2}$

$= -\frac{2}{x^2(1+\ln x)^3}+\frac{-1}{(1+\ln x)^2x^2}$

$= -\frac{2}{x^2(1+\ln x)^3}-\frac{(1+\ln x)}{(1+\ln x)^3x^2}$

$= \frac{-2}{x^2(1+\ln x)^3}+\frac{-1-\ln x}{(1+\ln x)^3x^2}$

$\boxed{= \frac{-\ln x-3}{x^2(1+\ln x)^3}}$

32. $\frac{\partial}{\partial x}\sqrt{2+\ln x}$

$= \frac{1}{2}(2+\ln x)^{-1/2}\cdot\frac{1}{x}$

$\boxed{= \frac{1}{2x\sqrt{2+\ln x}}}$

$2+\ln x \geq 0 \qquad x>0$

$\ln x \geq -2$

$x \geq e^{-2}$

$\boxed{x\in[e^{-2},\infty)}$

36. $\frac{d}{dx}\cos(\ln x^2)$

$= -\sin(\ln x^2)\frac{d}{dx}\ln x^2$

$= -\sin(\ln x^2)\frac{1}{x^2}\cdot 2x$

$= -\frac{\sin(\ln x^2)\cdot 2}{x}$

$= \frac{-2\sin(\ln x^2)}{x}$

$f'(1) = -2\sin(\ln 1) = -2\sin 0 = \boxed{0}$

42. $\frac{d}{dx}\log_b(3x^2-2)$

$= \frac{1}{\ln(b)(3x^2-2)}\cdot\frac{d}{dx}(3x^2-2)$

$= \frac{1}{\ln(b)(3x^2-2)}\ 6x$

$= \frac{6x}{\ln(b)(3x^2-2)}$

$\frac{6(1)}{\ln(b)(3(1)^2-2)} = 3$

$\ln(b)$

$\frac{6}{\ln(b)} = 3$

$\ln(b) = 2$

$\boxed{b = e^2}$

50. $y = (\sqrt{x})^x$

$\ln y = \ln\left[(\sqrt{x})^x\right]$

$\ln y = x\ln\sqrt{x}$

$\frac{d}{dx}\ln y = \frac{d}{dx}\left[x\ln\sqrt{x}\right]$

$\frac{1}{y}\cdot\frac{dy}{dx} = x\frac{d}{dx}\ln\sqrt{x} + \ln\sqrt{x}$

$\frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} + \ln\sqrt{x}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2} + \ln\sqrt{x}$

$\frac{dy}{dx} = y\left(\frac{1}{2} + \ln\sqrt{x}\right)$

$\boxed{\frac{dy}{dx} = (\sqrt{x})^x\left(\frac{1}{2} + \ln\sqrt{x}\right)}$

54. $y = (\ln x)^{\cos x}$

$\ln y = \ln\left[(\ln x)^{\cos x}\right]$

$\ln y = \cos x\cdot\ln(\ln x)$

$\frac{d}{dx}\ln y = \frac{d}{dx}\left[\cos x\cdot\ln(\ln x)\right]$

$\frac{1}{y}\cdot\frac{dy}{dx} = \cos x\cdot\frac{d}{dx}\ln(\ln x) + \ln(\ln x)\frac{d}{dx}\cos x$

$= \cos x\cdot\frac{1}{\ln x}\ \frac{1}{x} - \ln(\ln x)\sin x$

$= \frac{\cos x}{\ln x\cdot x} - \ln(\ln x)\sin x$

$\frac{dy}{dx} = y\left[\frac{\cos x}{x\ln x} - \ln(\ln x)\sin x\right]$

$\boxed{= (\ln x)^{\cos x}\left[\frac{\cos x}{x\ln x} - \ln(\ln x)\sin x\right]}$

56. $x^y = y^x$

$\ln(x^y) = \ln(y^x)$

$y \ln x = x \ln y$

$\frac{d}{dx}\left[y \ln x\right] = \frac{d}{dx}\left[x \ln y\right]$

$y \cdot \frac{1}{x} + \ln x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \ln y$

$\ln x \frac{dy}{dx} - \frac{x}{y} \frac{dy}{dx} = \ln y - \frac{y}{x}$

$\frac{dy}{dx} = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$

$= \frac{xy \ln y - y^2}{xy \ln x - x^2}$