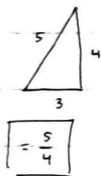


AP Calc AB: HW 6.6

$$\begin{aligned} \text{b. a. } \tan^{-1}(\tan \frac{3\pi}{4}) & \\ &= \tan^{-1}(-\tan \frac{\pi}{4}) \\ &= -\tan^{-1}(\tan \frac{\pi}{4}) \\ &= \boxed{-\frac{\pi}{4}} \end{aligned}$$

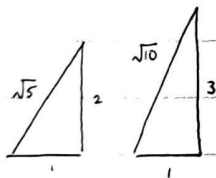
$$\begin{aligned} \text{b. } \cos(\sin^{-1} \frac{1}{2}) & \\ &= \cos \frac{\pi}{6} \\ &= \boxed{\frac{\sqrt{3}}{2}} \end{aligned}$$

$$\text{c. } \csc(\cos^{-1} \frac{3}{5})$$



$$10. \cos(\tan^{-1} 2 + \tan^{-1} 3)$$

$$= \cos(\tan^{-1} 2) \cos(\tan^{-1} 3) - \sin(\tan^{-1} 2) \sin(\tan^{-1} 3)$$



$$\begin{aligned} &= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right) - \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) \\ &= \frac{1-6}{\sqrt{50}} \\ &= -\frac{5}{\sqrt{50}} \\ &= -\frac{5}{5\sqrt{2}} \\ &= \boxed{-\frac{\sqrt{2}}{2}} \end{aligned}$$

$$\begin{aligned} 14. \sin(2 \cos^{-1} x) & \\ &= 2 \sin(\cos^{-1} x) \cos(\cos^{-1} x) \\ &= 2 \sin(\cos^{-1} x) \cdot x \\ \theta &= \cos^{-1} x \\ \cos \theta &= x \end{aligned}$$

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \sin^2 \theta &= 1 - \cos^2 \theta \\ \sin \theta &= \sqrt{1 - \cos^2 \theta} \\ &= \sqrt{1 - x^2} \end{aligned}$$

$$= 2 \sin \theta \cdot x$$

$$= \boxed{2x \sqrt{1-x^2}}$$

$$\begin{aligned} 22. \frac{d}{dx} \tan^{-1}(x^2) & \\ &= \frac{1}{1+x^4} \cdot 2x \\ &= \boxed{\frac{2x}{1+x^4}} \end{aligned}$$

$$\begin{aligned} 24. \frac{d}{dx} \cos^{-1} \sqrt{x} & \\ &= -\frac{1}{\sqrt{1-x}} \cdot \frac{d}{dx} \sqrt{x} \\ &= -\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \\ &= \boxed{-\frac{1}{2\sqrt{x}\sqrt{1-x}}} \end{aligned}$$

$$\begin{aligned} 26. \frac{d}{dt} \sin^{-1}\left(\frac{1}{t}\right) & \\ &= \frac{1}{\sqrt{1-\left(\frac{1}{t}\right)^2}} \cdot \frac{d}{dt} \left(\frac{1}{t}\right) \\ &= \frac{1}{\sqrt{1-\left(\frac{1}{t}\right)^2}} \cdot -t^{-2} \\ &= -\frac{1}{t^2 \sqrt{1-\left(\frac{1}{t}\right)^2}} \\ &= -\frac{1}{\sqrt{t^2-1}} \\ &= \boxed{-\frac{1}{t \sqrt{t^2-1}}} \end{aligned}$$

$$28. \frac{d}{dt} \cos^{-1}(\sin^{-1} t)$$

$$\begin{aligned} &= -\frac{1}{\sqrt{1-(\sin^{-1} t)^2}} \cdot \frac{d}{dt} \sin^{-1} t \\ &= -\frac{1}{\sqrt{1-(\sin^{-1} t)^2}} \cdot \frac{1}{\sqrt{1-t^2}} \end{aligned}$$

$$= \boxed{-\frac{1}{\sqrt{1-(\sin^{-1} t)^2} \cdot \sqrt{1-t^2}}}$$

$$30. \frac{d}{dx} \tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

$$\begin{aligned} &= \frac{1}{1+\frac{1-x}{1+x}} \cdot \frac{d}{dx} \sqrt{\frac{1-x}{1+x}} \\ &= \frac{1+x}{2} \cdot \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{d}{dx} \frac{1-x}{1+x} \\ &= \frac{x+1}{4} \cdot \left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{-(1+x) - (1-x)}{(1+x)^2} \end{aligned}$$

$$\begin{aligned} &= \frac{x+1}{4} \cdot \left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{-2}{(1+x)^2} \\ &= \frac{-1}{2(x+1)\sqrt{\frac{1-x}{1+x}}} \end{aligned}$$

$$= \boxed{-\frac{1}{2(x+1)\sqrt{\frac{1-x}{1+x}}}}$$

$$32. \frac{d}{dx} \tan^{-1}(x - \sqrt{1+x^2})$$

$$= \frac{1}{1 + (x - \sqrt{1+x^2})^2} \cdot \frac{d}{dx} (x - \sqrt{1+x^2})$$

$$= \frac{1}{1 + (x - \sqrt{1+x^2})^2} \cdot \left(1 - \frac{1}{2} (1+x^2)^{-1/2} \cdot 2x\right)$$

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$$= \frac{1 - x(1+x^2)^{-1/2}}{1 + (x - \sqrt{1+x^2})^2}$$

$$34. \frac{d}{dx} \left[\tan^{-1}\left(\frac{x}{a}\right) + \ln \sqrt{\frac{x-a}{x+a}} \right]$$

$$= \frac{1}{1 + \left(\frac{x}{a}\right)^2} \cdot \frac{1}{a} + \frac{1}{\sqrt{\frac{x-a}{x+a}}} \cdot \frac{-(x+a) - (x-a)}{(x+a)^2}$$

$$= \frac{a^2}{a^2+x^2} \cdot \frac{1}{a} + \left(\frac{x-a}{x+a}\right)^{-1/2} \cdot \frac{-2x}{(x+a)^2}$$

$$= \frac{a}{a^2+x^2} + \frac{\sqrt{x+a}}{\sqrt{x-a}} \cdot \frac{-2x}{(x+a)^2}$$

$$= \frac{1}{1 + \left(\frac{x}{a}\right)^2} \cdot \frac{1}{a} + \frac{d}{dx} \ln \sqrt{\frac{x-a}{x+a}}$$

$$= \frac{a^2}{a^2+x^2} \cdot \frac{1}{a} + \frac{d}{dx} \ln \sqrt{\frac{x-a}{x+a}}$$

$$= \frac{a}{a^2+x^2} + \frac{d}{dx} \ln \sqrt{\frac{x-a}{x+a}}$$

$$= \frac{a}{a^2+x^2} + \frac{1}{\sqrt{\frac{x-a}{x+a}}} \cdot \frac{d}{dx} \sqrt{\frac{x-a}{x+a}}$$

$$= \frac{a}{a^2+x^2} + \frac{1}{\sqrt{\frac{x-a}{x+a}}} \cdot \frac{1}{2} \left(\frac{x-a}{x+a}\right)^{-1/2} \cdot \frac{(x+a) - (x-a)}{(x+a)^2}$$

$$= \frac{a}{a^2+x^2} + \frac{1}{\sqrt{\frac{x-a}{x+a}}} \cdot \frac{1}{2} \left(\frac{x+a}{x-a}\right)^{1/2} \cdot \frac{2a}{(x+a)^2}$$

$$= \frac{a}{a^2+x^2} + \frac{x+a}{x-a} \cdot \frac{a}{(x+a)^2}$$

$$= \frac{a}{a^2+x^2} + \frac{a}{(x-a)(x+a)}$$

$$40. \frac{d}{dx} 3 \cos^{-1}\left(\frac{x}{2}\right)$$

$$= 3 \cdot -\frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}$$

$$= -\frac{3}{2\sqrt{1 - \left(\frac{x}{2}\right)^2}}$$

$$= -\frac{3}{2 \cdot \sqrt{\frac{3}{4}}}$$

~~scribble~~

$$= -\frac{3}{2 \cdot \frac{\sqrt{3}}{2}} = -\frac{3\sqrt{3}}{3} = -\sqrt{3}$$

~~scribble~~

$$y - \pi = -\sqrt{3}(x-1)$$

$$44. \lim_{x \rightarrow \infty} \cos^{-1}\left(\frac{1+x^2}{1+2x^2}\right)$$

$$= \cos^{-1}\left(\lim_{x \rightarrow \infty} \frac{1+x^2}{1+2x^2}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$