James Ding

# AP Calc AB : HW 6.6

6. a. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$= -\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

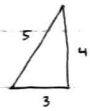
$\boxed{= -\frac{\pi}{4}}$

b. $\cos\left(\sin^{-1}\frac{1}{2}\right)$

$= \cos\frac{\pi}{6}$

$\boxed{= \frac{\sqrt{3}}{2}}$

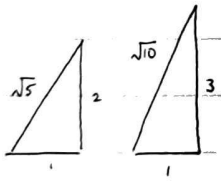
8. $\csc\left(\cos^{-1}\frac{3}{5}\right)$




$\boxed{= \frac{5}{4}}$

10. $\cos(\tan^{-1}2 + \tan^{-1}3)$

$= \cos(\tan^{-1}2)\cos(\tan^{-1}3) - \sin(\tan^{-1}2)\sin(\tan^{-1}3)$




$= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right) - \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right)$

$= \frac{1-6}{\sqrt{50}}$

$= -\frac{5}{\sqrt{50}}$

$= -\frac{5}{5\sqrt{2}}$

$\boxed{= -\frac{\sqrt{2}}{2}}$

14. $\sin(2\cos^{-1}x)$

$= 2\sin(\cos^{-1}x)\cos(\cos^{-1}x)$

$= 2\sin(\cos^{-1}x)\cdot x$

$\theta = \cos^{-1}x$

$\cos\theta = x$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-x^2}$

$= 2\sin\theta \cdot x$

$\boxed{= 2x\sqrt{1-x^2}}$

22. $\frac{d}{dx}\tan^{-1}(x^2)$

$= \frac{1}{1+x^4}\cdot 2x$

$\boxed{= \frac{2x}{1+x^4}}$

24. $\frac{d}{dx}\cos^{-1}\sqrt{x}$

$= -\frac{1}{\sqrt{1-x}}\cdot\frac{d}{dx}\sqrt{x}$

$= -\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$

$\boxed{= -\frac{1}{2\sqrt{x}\sqrt{1-x}}}$

26. $\frac{d}{dt}\sin^{-1}\left(\frac{1}{t}\right)$

$= \frac{1}{\sqrt{1-\left(\frac{1}{t}\right)^2}}\cdot\frac{d}{dt}\left(\frac{1}{t}\right)$

$= \frac{1}{\sqrt{1-\left(\frac{1}{t}\right)^2}}\cdot -t^{-2}$

$= -\frac{1}{t^2\sqrt{1-\left(\frac{1}{t}\right)^2}}$

$= -\frac{1}{\sqrt{t^4-t^2}}$

$\boxed{= -\frac{1}{|t|\sqrt{t^2-1}}}$

28. $\frac{d}{dt}\cos^{-1}(\sin^{-1}t)$

$= -\frac{1}{\sqrt{1-(\sin^{-1}t)^2}}\cdot\frac{d}{dt}\sin^{-1}t$

$= -\frac{1}{\sqrt{1-(\sin^{-1}t)^2}}\cdot\frac{1}{\sqrt{1-x^2}}$

$\boxed{= -\frac{1}{\sqrt{1-(\sin^{-1}t)^2}\cdot\sqrt{1-x^2}}}$

30. $\frac{d}{dx}\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$

$= \frac{1}{1+\frac{1-x}{1+x}}\cdot\frac{d}{dx}\sqrt{\frac{1-x}{1+x}}$

$= \frac{1+x}{2}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\cdot\frac{d}{dx}\frac{1-x}{1+x}$

$= \frac{x+1}{4}\cdot\left(\frac{1-x}{1+x}\right)^{-1/2}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}$

$= \frac{x+1}{4}\cdot\left(\frac{1-x}{1+x}\right)^{-1/2}\cdot\frac{-2}{(1+x)^2}$

$\boxed{= \frac{-1}{2(x+1)\sqrt{\frac{-(x-1)}{x+1}}}}$

32. $\frac{d}{dx}\tan^{-1}\left(x-\sqrt{1+x^2}\right)$

$= \frac{1}{1+\left(x-\sqrt{1+x^2}\right)^2}\cdot\frac{d}{dx}\left(x-\sqrt{1+x^2}\right)$

$= \frac{1}{1+\left(x-\sqrt{1+x^2}\right)^2}\cdot\left(1-\frac{1}{2}\left(1+x^2\right)^{-1/2}\cdot 2x\right)$

$= \boxed{\frac{1-x\left(1+x^2\right)^{-1/2}}{1+\left(x-\sqrt{1+x^2}\right)^2}}$

34. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{a}\right)+\ln\sqrt{\frac{x-a}{x+a}}\right]$

$= \frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1}{a}+\frac{1}{\sqrt{\frac{x-a}{x+a}}}\cdot\frac{-(x+a)-(x-a)}{(x+a)^2}$

$= \frac{a^2}{a^2+x^2}\cdot\frac{1}{a}+\left(\frac{x-a}{x+a}\right)^{-1/2}\cdot\frac{-2x}{(x+a)^2}$

$= \frac{a}{a^2+x^2}+\frac{\sqrt{x+a}}{\sqrt{x-a}}\cdot\frac{-2x}{(x+a)^2}$

$= \frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1}{a}+\frac{d}{dx}\ln\sqrt{\frac{x-a}{x+a}}$

$= \frac{a^2}{a^2+x^2}\cdot\frac{1}{a}+\frac{d}{dx}\ln\sqrt{\frac{x-a}{x+a}}$

$= \frac{a}{a^2+x^2}+\frac{d}{dx}\ln\sqrt{\frac{x-a}{x+a}}$

$= \frac{a}{a^2+x^2}+\frac{1}{\sqrt{\frac{x-a}{x+a}}}\cdot\frac{d}{dx}\sqrt{\frac{x-a}{x+a}}$

$= \frac{a}{a^2+x^2}+\frac{1}{\sqrt{\frac{x-a}{x+a}}}\cdot\frac{1}{2}\left(\frac{x-a}{x+a}\right)^{-1/2}\cdot\frac{(x+a)-(x-a)}{(x+a)^2}$

$= \frac{a}{a^2+x^2}+\frac{1}{\sqrt{\frac{x-a}{x+a}}}\cdot\frac{1}{2}\left(\frac{1}{\sqrt{\frac{x-a}{x+a}}}\right)^{1}\cdot\frac{2a}{(x+a)^2}$

$= \frac{a}{a^2+x^2}+\frac{x+a}{x-a}\cdot\frac{a}{(x+a)^2}$

$= \boxed{\frac{a}{a^2+x^2}+\frac{a}{(x-a)(x+a)}}$

40. $\frac{d}{dx}\,3\cos^{-1}\left(\frac{x}{2}\right)$

$= 3\cdot-\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\frac{1}{2}$

$= -\frac{3}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}$

$-\frac{3}{2\cdot\sqrt{\frac{3}{4}}}$

$= -\frac{3}{2\cdot\frac{\sqrt{3}}{2}} = -\frac{3\sqrt{3}}{3} = -\sqrt{3}$

$\boxed{y-\pi = -\sqrt{3}(x-1)}$

44. $\lim_{x\to\infty}\cos^{-1}\left(\frac{1+x^2}{1+2x^2}\right)$

$= \cos^{-1}\left(\lim_{x\to\infty}\frac{1+x^2}{1+2x^2}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right)$

$= \boxed{\frac{\pi}{3}}$