HW #1

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Math N1A: Alg & Precal Review

1. a. • Seeing is always easier than doing it yourself.
• Practicing something new will inevitably lead to you making mistakes.

b. Visit OH, SLC, classmates, and Piazza

2. a. $-4^2 - 1^4$
$= -16 - 1$
$\boxed{= -17}$

b. $\sqrt{13^2 - 5^2}$
$= \sqrt{169 - 25}$
$= \sqrt{144}$
$\boxed{= 12}$

c. $(\sqrt{13} - \sqrt{5})^2$
$= (\sqrt{13})^2 - 2\sqrt{13}\cdot\sqrt{5} + (\sqrt{5})^2$
$= 13 - 2\sqrt{13}\cdot\sqrt{5} + 5$
$\boxed{= 18 - 2\sqrt{65}}$

3. a. $16^{-3/2}$
$= (16^{\frac{1}{2}})^{-3}$
$= 4^{-3}$
$= \frac{1}{4^3}$
$\boxed{= 1/64}$

b. $\sqrt[5]{27^{3/2}\sqrt{3}}$
$= (27^{3/2}\sqrt{3})^{1/5}$
$= 27^{\frac{3}{2}\cdot\frac{1}{5}}\ 3^{\frac{1}{2}\cdot\frac{1}{5}}$
$= 27^{3/10}\ 3^{1/10}$
$= \sqrt[10]{27^3 \cdot 3}$
$= \sqrt[10]{3^9 \cdot 3}$
$\boxed{= 3}$

4. a. $\log_3 9$
$\Rightarrow 3^x = 9$
$x = 2$
$\boxed{\Leftarrow 2}$

b. $\log_\pi 0$
$\boxed{\text{DNE: } \log_a b,\ b > 0}$

c. cont'd.

4. c. $\log_8\left(\frac{1}{\sqrt[7]{32}}\right)$
$= \log_8 32^{-\frac{1}{7}}$
$= -\frac{1}{7}\log_8 32$
$= -\frac{1}{7}\log_8 2^5$
$= -\frac{5}{7}\log_8 2$
$\Rightarrow 8^x = 2$
$x = \frac{1}{3}$ (cube root)
$\Leftarrow -\frac{5}{7}\cdot\frac{1}{3}$
$\boxed{= -\frac{5}{21}}$

5. a. $\log_6 24 + \log_6 3 - \log_6 2$
$= \log_6 72 - \log_6 2$
$= \log_6 36$
$\Rightarrow 6^x = 36$
$x = 2$
$\boxed{\Leftarrow 2}$

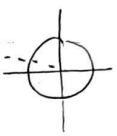
b. $2\ln 4 - 4\ln 2$
$= \ln 16 - \ln 16$
$\boxed{= 0}$

c. $\log_4 49 + \log_2 \frac{8}{7}$
$= \log_4 49 + \frac{\log_4 \frac{8}{7}}{\log_4 2}$
$= \log_4 49 + \frac{\log_4 \frac{8}{7}}{1/2}$
$= \log_4 49 + 2\log_4 \frac{8}{7}$
$= \log_4 49 + \log_4 \frac{64}{49}$
$= \log_4 49 + \log_4 64 - \log_4 49$
$= \log_4 64$
$\Rightarrow 4^x = 64$
$x = 3$
$\boxed{\Leftarrow 3}$

6. a. $\cos 225°$
$= -\cos 45°$
$\boxed{= -\frac{\sqrt{2}}{2}}$

b. $\tan(-3\pi/2)$
$= \tan\frac{\pi}{2}$
$\boxed{= \text{DNE}}$

c. $\sin\left(\frac{17\pi}{6}\right)$
$= \sin\left(\frac{5\pi}{6}\right)$
$= \sin\frac{\pi}{6}$
$\boxed{= \frac{1}{2}}$



d. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$
$= \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
$\boxed{= \frac{3\pi}{4}}$

7. a. $\boxed{10.85}$

b. $\boxed{8.1 + e}$
(Gives 10.8183...)

8. a. $p^5 + 3p^4 + 2p^3$
$= p^3(p^2 + 3p + 2)$
$\boxed{= p^3(p+1)(p+2)}$

b. $p^8 - 16q^4r^{12}$
$= (p^4)^2 - ((2qr^3)^2)^2$
$= (p^4 + (2qr^3)^2)(p^4 + -(2qr^3)^2)$
$\boxed{= (p^4 + 4q^2r^6)(p^2 + 2qr^3)(p^2 - 2qr^3)}$

9. $$\frac{(x+3)^2(3)(7-4x)^2(-4)-(7-4x)^3(2)(x+3)}{((x+3)^2)^2}$$

$$= \frac{-12(x+3)^2(7-4x)^2-2(7-4x)^3(x+3)}{(x+3)^4}$$

$$= \frac{-2(x+3)(7-4x)^2\left(6(x+3)+(7-4x)\right)}{(x+3)^4}$$

$$= \frac{-2(7-4x)^2(6x+18+7-4x)}{(x+3)^3}$$

$$\boxed{= \frac{-2(7-4x)^2(2x+25)}{(x+3)^3}}$$

10. $$\frac{g(1+h)-g(1)}{h}$$

$$= \frac{\frac{5}{1+h}-5}{h}$$

$$\Rightarrow \frac{5}{1+h} - \frac{5(1+h)}{1+h}$$

$$= \frac{5-5-5h}{1+h}$$

$$\Leftarrow \frac{\frac{-5h}{1+h}}{h}$$

$$= \frac{-5h}{h(1+h)}$$

$$\boxed{= \frac{-5}{h+1} \; ; \; x \neq 0, -1}$$

11. $f(2) = 2^3 - 2(2)$

$= 8-4$

$= 4$

$f(2^{-1}) = \left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)$

$= \frac{1}{8} - 1$

$= -\frac{7}{8}$

$f(2) - f(2^{-1})$

$= 4 + \frac{7}{8}$

$$\boxed{= \frac{39}{8}}$$

12. $$\frac{f(b)-f(3a)}{b-3a}$$

$$= \frac{(b^3-2b)-((3a)^3-2(3a))}{b-3a}$$

$$= \frac{b^3-2b-27a^3+6a}{b-3a}$$

$$= \frac{b^3-27a^3-2b+6a}{b-3a}$$

$$= \frac{(b-3a)(b^2+3ab+9a^2)-2(b-3a)}{b-3a}$$

$$\boxed{= b^2+3ab+9a^2-2 \; ; \; b \neq 3a}$$

13. a. $t^2 - t - 6 = 0$

$= (t-3)(t+2)$

$\boxed{= 3, -2}$

b. $t^2 - t + 6 = 0$

$$= \frac{1 \pm \sqrt{1-24}}{2}$$

$\Rightarrow 1 - 24 < 0$; No real answers

$\boxed{= \text{ONE}}$

14. a. $(z^4-2)^2 - 3 = 0$

$= z^8 - 4z^4 + 4 - 3$

$= z^8 - 4z^4 + 1$

let $u = z^4$

$\Rightarrow u^2 - 4u + 1$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$= 2 \pm \sqrt{3}$

$\Leftarrow z^4 = 2 \pm \sqrt{3}$

$z = \pm\sqrt[4]{2 \pm \sqrt{3}}$

$$\boxed{= -\sqrt[4]{2-\sqrt{3}}, -\sqrt[4]{2+\sqrt{3}}, \sqrt[4]{2-\sqrt{3}}, \sqrt[4]{2+\sqrt{3}}}$$

No extraneous ✓

b. $(z^4-2)^2 - 10 = 0$

$= z^8 - 4z^4 + 4 - 10$

$= z^8 - 4z^4 - 6$

$\Rightarrow$ let $u = z^4$

$= u^2 - 4u - 6$

$$= \frac{4 \pm \sqrt{40}}{2}$$

$$= \frac{4 \pm 2\sqrt{10}}{2}$$

$= 2 \pm \sqrt{10}$

$\Leftarrow z^4 = 2 \pm \sqrt{10}$

$z = \pm\sqrt[4]{2 \pm \sqrt{10}}$

$= -\sqrt[4]{2-\sqrt{10}}, -\sqrt[4]{2+\sqrt{10}}, \sqrt[4]{2-\sqrt{10}}, \sqrt[4]{2+\sqrt{10}}$

↓ Ext. ↓ Ext.

$$\boxed{= -\sqrt[4]{2+\sqrt{10}}, \sqrt[4]{2+\sqrt{10}}}$$

15. $|3x-4| - 5x = 12$

$|3x-4| = 5x+12$

$3x-4 = 5x+12$ $\quad -3x+4 = 5x+12$

$3x = 5x+16$ $\quad -3x = 5x+8$

$-2x = 16$ $\quad -8x = 8$

$x = -8$ $\quad \boxed{x = -1}$

Extraneous

16. a. $27^{1-2t} = 9$

$= (3^3)^{1-2t} = 3^2$

$\Rightarrow 3(1-2t) = 2$

$= 3-6t = 2$

$= -6t = -1$

$t = \frac{1}{6}$

$\boxed{\Leftarrow \frac{1}{6}}$

b. $\log_3(y^3-2y^2-9y+27) - 2 = 0$

$= \log_3(y^3-2y^2-9y+27) = 2$

$\Rightarrow 3^2 = y^3-2y^2-9y+27$

$\Leftarrow 9 = y^3-2y^2-9y+27$

$0 = y^3-2y^2-9y+18$

$= y^2(y-2) - 9(y-2)$

$= (y^2-9)(y-2)$

$= (y-3)(y+3)(y-2)$

$\boxed{y = -3, 2, 3}$

No extraneous

17. $x^6+7x^3-8 = 0$

let $u = x^3$

$\Rightarrow u^2+7u-8 = 0$

$= (u+8)(u-1) = 0$

$\Leftarrow (x^3+8)(x^3-1) = 0$

$= (x+2)(x^2-2x+4)(x-1)(x^2+x+1)$

↓ Ext. ↓ Ext.

$\boxed{= -2, 1}$

18. $2\sin^2 x - \sin x = 1$

$= 2\sin^2 x - \sin x - 1 = 0$

let $u = \sin x$

$\Rightarrow 2u - u - 1 = 0$

$= \frac{1 \pm \sqrt{9}}{4}$

$= \frac{1 \pm 3}{4}$

$= 1, -\frac{1}{2}$

$\sin x = 1$ $\quad \sin x = -\frac{1}{2}$

a. $\sin x = 1$; $0 \le x \le 2\pi$ $\quad \sin x = -\frac{1}{2}$; $0 \le x \le 2\pi$

$x = \frac{\pi}{2}$ $\quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\boxed{x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

b. $\sin x = 1$ $\quad \sin x = -\frac{1}{2}$

$x = \frac{\pi}{2} + 2\pi k$ $\quad x = \frac{7\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k$

$\boxed{x = \frac{\pi}{2} + 2\pi k, \frac{7\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k}$

(let k be any integer)

19. $\frac{x+3}{x^3-9x}$ (Worked w/ Santiago)

$x^3 - 9x \neq 0$

$= x(x^2-9)$

$= x(x-3)(x+3)$

$x \neq 0, 3, -3$

$\boxed{(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)}$

20. $\frac{x+3}{x^3-9x}$

$x^3 - 9x \neq 0$

$x \neq 0, 3, -3$ (from #19) $\quad \frac{(x+3)}{x(x-3)(x+3)} \ge 0$

$= \frac{1}{x(x-3)} \ge 0$

Cutpoints: $-3, 0, 3$

Interval	Result
$(-\infty, -3)$	+
$(-3, 0)$	+
$(0, 3)$	− ← Invalid interval
$(3, \infty)$	+

$\boxed{(-\infty, 3) \cup (-3, 0) \cup (3, \infty)}$