HW # 10

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Math N1A: Sections 4.4, 4.5, 4.7

Section 4.4

46. $\lim_{x\to-\infty} x\ln\left(1-\frac{1}{x}\right)$

$= \lim_{x\to-\infty} \frac{\ln\left(1-\frac{1}{x}\right)}{\frac{1}{x}} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to-\infty} \frac{\frac{1}{1-\frac{1}{x}}\cdot(x^{-2})}{-x^{-2}}$

$= \lim_{x\to-\infty} \frac{\frac{1}{x^2\left(1-\frac{1}{x}\right)}}{-x^{-2}}$

$= \lim_{x\to-\infty} \frac{-x^2}{x^2\left(1-\frac{1}{x}\right)}$

$= \lim_{x\to-\infty} -\frac{1}{1-\frac{1}{x}}$

$\boxed{=-1}$

51. $\lim_{x\to1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) \Rightarrow \infty-\infty$ (Solve via common denom.)

$= \lim_{x\to1} \frac{x\ln x-(x-1)}{\ln x(x-1)} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to1} \frac{x\left(\frac{1}{x}\right)+\ln x-1}{\ln x+x\left(\frac{1}{x}\right)-\frac{1}{x}}$

$= \lim_{x\to1} \frac{1+\ln x-1}{\ln x+1-\frac{1}{x}}$

$= \lim_{x\to1} \frac{\ln x}{\ln x+1-\frac{1}{x}} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to1} \frac{\frac{1}{x}}{\frac{1}{x}+x^{-2}}$

$= \lim_{x\to1} \frac{1}{x\left(\frac{1}{x}+\frac{1}{x^2}\right)}$

$= \lim_{x\to1} \frac{1}{1+\frac{1}{x}}$

$\boxed{=\frac{1}{2}}$

13. A. Domain: $\boxed{\text{All } \mathbb{R} \text{ except } \pm2}$

B. Y-intercept: $\frac{0}{0^2-4}=\boxed{0}$

x-intercept: $0=\frac{x}{x^2-4}$; $\boxed{x=0}$

C. $f(-x)=\frac{-x}{(-x)^2-4}=-f(x)$ $\therefore$ $\boxed{\text{Function is odd}}$

D. Asymptotes

Vertical Asymptotes exist at $x=\pm2$, as they are unable to be simplified.

$\lim_{x\to-\infty}\frac{x}{x^2-4} \Rightarrow \frac{x}{x^2\left(1-\frac{4}{x^2}\right)} \Rightarrow \frac{1}{x\left(1-\frac{4}{x^2}\right)}=0$

$\lim_{x\to\infty}\frac{x}{x^2-4} \Rightarrow \frac{x}{x^2\left(1-\frac{4}{x^2}\right)} \Rightarrow \frac{1}{x\left(1-\frac{4}{x^2}\right)}=0$

$\boxed{\therefore \text{ Horizontal Asymptote at } y=0}$

E. $f'(x)=\frac{d}{dx}\frac{x}{x^2-4}$

$=\frac{(x^2-4)-x(2x)}{(x^2-4)^2}$

$=\frac{-(x^2+4)}{(x+2)^2(x-2)^2}$

Always negative, as each "element" is always > 0. (i.e always decreasing)

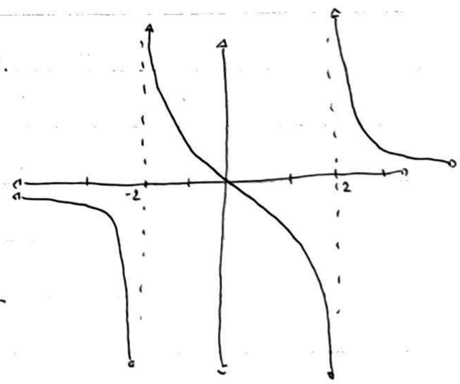
F. No Critical points.

G. $f''(x)=\frac{d}{dx}\left[\frac{-(x^2+4)}{(x+2)^2(x-2)^2}\right]$

$=\frac{2x(x^2+12)}{(x^2-4)^3}$

| | $-2$ | | $0$ | | $2$ | |
|---|---|---|---|---|---|---|
| $2x$ | $-$ | | $-$ | $+$ | | $+$ |
| $(x^2-4)^3$ | $+$ | | $-$ | $-$ | | $+$ |
| $f''(x)$ | $-$ | $+$ | | $-$ | | $+$ |
| Concavity: | down | up | | down | | up. |

H.



## Section 4.7

3. $ab = 100$

$s = a + b$

This is a min!

$- \quad 10 \quad +$

$a = 100/b$

$s = \frac{100}{b} + b$

$\frac{d}{db}\left[\frac{100}{b} + b\right]$

$= 1 - \frac{100}{b^2}$

$0 = 1 - \frac{100}{b^2}$ ; ~~b~~ $b \in (0, \infty)$

$b = 10$ ; $b \neq 0$ as not within domain, and $a \cdot (0)$ will never yield $100$

$a(10) = 100$

$a = 10$

$\boxed{a = 10, b = 10}$

b. To solve for max, we find the absolute max value of $x^2 + (5-x)^2$, when $x \in [0, 5]$ (there can be degenerate pens, which is given)

Max side value must be 5 ; $4x = 20$, $x = 5$

From a, we have discoved the only critical point at $x^2 + (5-x)^2$ occurs when $x = \frac{5}{2}$, yielding $\frac{25}{2} m^2$.

Lets take the endpoints of the interval

$(0)^2 - (5-0)^2 = 25$

$(5)^2 - (5-5)^2 = 25$

Both endpoints yield the same area.

$\therefore$ This tells us only having 1 pen yields the most area with 20m of fencing

$\boxed{\text{Max: } 25m^2}$

## Luvveet's Problem S

a. $s = x^2 + y^2$

$x \quad y$

$20 = 4x + 4y$

$4(x+y) = 20$

$x + y = 5$

$y = (5 - x)$

$s = x^2 + (5-x)^2 = 2x^2 - 10x + 25$

Quadratic, which follows $\Psi$ shape

$\frac{d}{dx}\left[x^2 + (5-x)^2\right]$

$= 2x + 2(5-x)(-1)$

$= 4x - 10$

Sign change must occur at x-intercept

$0 = 4x - 10$

$10 = 4x$

$x = \frac{5}{2}$

$s = \left(\frac{5}{2}\right)^2 + \left(5 - \frac{5}{2}\right)^2$

$= \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2$

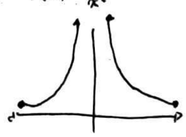
$= \frac{25}{4} + \frac{25}{4}$

$\boxed{= \frac{25}{2}} m^2$

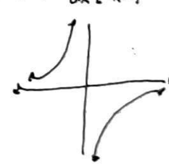
## Other Problems

1. $\boxed{\text{False}}$ Look at $\frac{1}{x^2}$

$f(x) = \frac{1}{x^2}$



$f'(x) = \frac{d}{dx}\left[\frac{1}{x^2}\right]$



Notice how a local max does not exist at $x = 0$

2. $\boxed{\text{True.}}$ That's the definition of a local max, provided the function is continuous at I and differentiable at I.

3. $\lim_{x \to 0} \frac{\ln(1-x) + x}{x^4} \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{\frac{1}{1-x} \cdot (-x) + 1}{4x^3}$

$= \lim_{x \to 0} \frac{-\frac{x}{1-x} + 1}{4x^3}$

$= \lim_{x \to 0} \frac{-\frac{x(1-x)}{1-x}}{4x^3}$

$= \lim_{x \to 0} \frac{1}{4x^3(1-x)} \Rightarrow \frac{1}{0}$

$\lim_{x \to 0^-} \frac{1}{4x^3(1-x)} = \frac{1}{0^-} = -\infty$

$\lim_{x \to 0^+} \frac{1}{4x^3(1-x)} = \frac{1}{0^+} = \infty$

$\boxed{\text{Limit doesn't exist}}$

4. $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x} \Rightarrow 1^{\infty}$

$y = \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$

$\ln(y) = \ln\left(\lim_{x\to\infty}\left(1+\frac{2}{x}\right)\cdot x\right)$

$= \lim_{x\to\infty}\left(\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}\right) \Rightarrow \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\frac{2}{x}}\cdot\left(-2x^{-2}\right)}{-x^{-2}}$

$= \lim_{x\to\infty}\frac{2}{1+\frac{2}{x}}$

$\ln(y) = 2$

$\boxed{e^2 = y}$

5. $\lim_{x\to\infty} x^{\frac{1}{x}}$

$y = \lim_{x\to\infty} x^{\frac{1}{x}}$

$\ln(y) = \ln\left(\lim_{x\to\infty} x^{\frac{1}{x}}\right)$

$= \lim_{x\to\infty}\left(\ln\left(x^{\frac{1}{x}}\right)\right)$

$= \lim_{x\to\infty}\frac{\ln(x)}{x} \Rightarrow \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty}\frac{\frac{1}{x}}{1}$

$= \lim_{x\to\infty}\frac{1}{x}$

$\ln(y) = 0$

$y = e^0 = 1$

$\boxed{y=1}$

6. $\lim_{x\to 0^+} x^{\frac{1}{x}} \Rightarrow 0^{\infty}$

$y = \lim_{x\to 0^+} x^{\frac{1}{x}}$

$\ln(y) = \ln\left(\lim_{x\to 0^+} x^{\frac{1}{x}}\right)$

$= \lim_{x\to 0^+}\frac{\ln(x)}{x}$

$\ln(y) = -\infty$

$e^{-\infty} = y$

$\boxed{y=0}$

7.

| | | -6 | | -3 | | -1 | | 4 | |
|---|---|---|---|---|---|---|---|---|---|
| $f'(x)$ | + | | − | | − | | − | | + |
| $f''(x)$ | − | | − | | + | | − | | − |